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THE

PHYSICAL REVIEW.

STUDIES OF THE LIME LIGHT.

By Edward L. Nichols and Mary L. Crehore.

I.

THE experiments to be described in this paper were made for the purpose of tracing the changes which the radiation from the hottest part of the incandescent lime cylinder used in the Drummond light undergoes from the moment of ignition until the lime has reached its final state of incandescence. It has long been known that the brilliancy of incandescent lime diminishes rapidly from the moment that the cylinder is brought under the action of the oxyhydrogen flame, but the law of its degeneration has not, so far as we are aware, been definitely studied. W. H. Pickering [1] in 1880 made some measurements of the lime light in its final condition. In 1889, one of the writers, in collaboration with Professor W. S. Franklin,[2] studied the spectrum of this source in the course of a series of investigations upon the quality of the light emitted by various artificial illuminants. In the latter research, freshly ignited cylinders, and also those which had been for a considerable time subjected to the temperature of the flame, were measured. It was found that the freshly ignited lime was whiter, that is to say, relatively stronger, in the shorter wave-lengths than the arc light or than any other artificial source

[1] Pickering, Proceedings of the American Academy of Arts and Sciences, Vol. 15.

[2] Nichols and Franklin, American Journal of Science, Vol. 38, p. 100.

excepting the magnesium light, to which it showed a very close resemblance. In its final state of incandescence, however, the lime light was found to fall decidedly below the electric arc in whiteness, being relatively about twice as bright in the blue as gas light. Measurements described in the paper just referred to showed that the spectrum of the lime in its final state of incandescence gave readings which corresponded closely to those obtained by Pickering in his investigation.

In the present paper it is proposed to trace more in detail these time changes, for which purpose the work was divided into two distinct portions. The first method pursued deals with the radiation from the lime taken as a whole, and with such portions of its radiation as are capable of being transmitted by water and

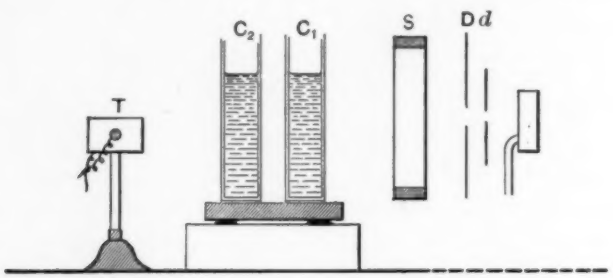

Fig. 1.

glass. Throughout the determinations, diaphragms were mounted in front of the lime cylinder, these being so placed as to isolate a small region of the radiating surface where the lime was at its highest temperature. These rays were allowed to fall upon the face of the thermopile, which was in closed circuit with a galvanometer of high sensitiveness. Fig. 1 shows the arrangement of the apparatus. Interposed between the thermopile *T*, and the diaphragms *D*, *d*, already described, was a double, opaque screen, *S*, which could be withdrawn by the observer at the telescope, thus allowing the rays from the incandescent cylinder to fall upon the face of the thermopile during a single swing of the galvanometer. The throw thus produced served as an indication of the intensity of total radiation from that portion of the surface of the cylinder which was visible from the face of the thermopile.

Upon a track between the thermopile and the lime light were mounted two cells, C_1 and C_2, containing water. These could be interposed in the path of the rays, thus cutting off the greater part of the non-luminous radiation. The deflection produced when these cells were interposed, when compared with that produced by the total radiation, afforded data for the computation of the radiant efficiency of the lime cylinder.

Total radiation and light-giving radiation were treated in these experiments as functions of the time, and their ratio, corrected for the amount of light which the cells absorbed, and for the amount of non-luminous energy which they transmitted, gave the radiant efficiency as a function of the time also. Fig. 2, which is from the average of three sets of measurements, shows the decadence of total radiation and of the radiation transmitted through the cells from time to time, as indicated by the falling off in galvanometer deflection under the two conditions already stated. It will be noted that the decadence in both cases is very marked. The deflection, for example, due to the uninterrupted rays, falls off in thirty minutes to one-third its initial value. The deflections for the rays transmitted by the water cells suffer similar loss.

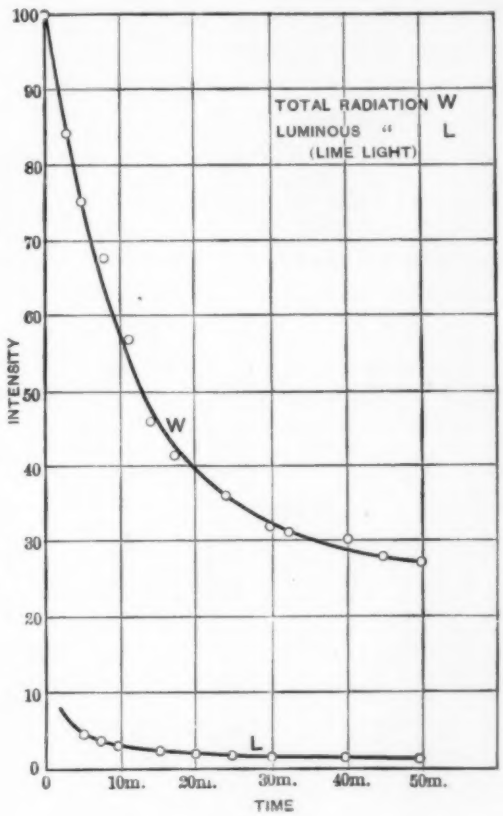

Fig. 2.

In order to compute from these two curves the radiant efficiency of the lime light, it is necessary to know how large a proportion of the light-giving radiation is transmitted by the water cells and also in how far these are impervious to the wave-lengths lying beyond the red. In nearly all previous studies of radiant heat, in which the method of Melloni has been used, it has been assumed that the proper material as regards opacity to such wave-lengths was a solution of alum in water. Recent observations by Mr. E. F. Nichols[1] upon the transmitting power of various substances in the infra-red have shown, however, that the addition of alum adds nothing to the adiathermancy of water, and that cells containing distilled water with no alum in solution will serve just as well for the purpose of obstructing the radiation of the infra-red as the alum solutions which have been used by so many observers. The precise relationship between the water and water containing ammonium and potassium and alums in solution is shown in the paper of Mr. E. F. Nichols already referred to. It will be seen from his diagram that these solutions all become opaque at the same point; viz. at wave-length 1.5. Also that they all reach about the same transparency at the boundary of the red, for which region they transmit about 80 per cent of the incident radiation.

It seems entirely justifiable in view of these results to use distilled water instead of a solution of alum for the purpose of this experiment. Measurements of the amount of light-giving energy transmitted by the two cells were made, and it was found that 55 per cent of the light incident upon the face of the first cell passed through both. An estimate of the proportion of the incident radiation of wave-lengths too long to affect the retina gave 0.004 as the transmitting power for such radiation.

By means of these two corrections it is a simple matter to compute the radiant efficiency of the lime light, which is given as a function of the time in Fig. 3. This curve shows that the radiant efficiency, which is the ratio of light-giving to total radiation, falls from about 14 per cent to a little more than 8 per cent during the first half-hour after the ignition of the lime.

It is interesting to note that these radiant efficiencies corre-

[1] Physical Review, Vol. I., p. 1.

spond very well with those which one would assign to the lime light upon the basis of our previous knowledge. It has already been pointed out that measurements of the spectrum of the freshly ignited lime showed a degree of incandescence corresponding closely with that of the magnesium light, while the spectrum of the lime in its final condition shows a state of incandescence somewhat slightly inferior to that of the electric arc. Mr. F. J. Rogers,[1] who has studied the magnesium lamp from the point of view of its radiant efficiency, finds, however, the value of the efficiency to be 0.137, a quantity which agrees very closely with the efficiency of the freshly ignited lime. The efficiency of the electric arc, on the other hand, according to numerous measurements, varies between 0.08 and 0.11, so that a comparison of the spectrum of lime in its final condition with the spectrum of the arc would lead us to a value for its final radiant efficiency, which is in excellent agreement with the values given in Fig. 3. Some measurements were made also upon the radiation of cylinders of magnesia and of zircon as a function of the time, and it was found that these sources, like lime, suffered marked decadence.

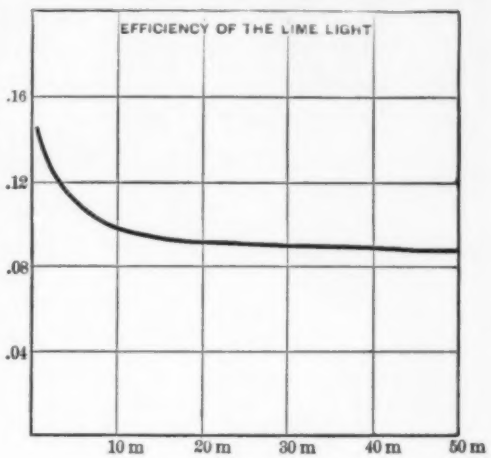

Fig. 3.

II.

The second portion of our investigation was made with the spectrophotometer, the object being to determine the law of decadence, wave-length by wave-length, throughout the visible spectrum. The arrangement of the apparatus is shown in Fig. 4.

[1] Rogers, American Journal of Science, Vol. 43, p. 301.

It consisted of a one-prism spectroscope provided with a Vierordt slit. This device for bringing the brightness of comparison spectra to a common value is the earliest and simplest of spectrophotometric devices. It is useful only where the range of intensity is small, but for the purpose of the present investigation it gives reliable results.

The lime cylinder was mounted a short distance before the slit of this spectroscope, two diaphragms, as in the first portion of the work, being interposed. The upper half of the slit was covered by means of a rectangular prism for total reflection, one of the acute edges of which formed the boundary between the two spectra. This edge was made to coincide as accurately as possible with the

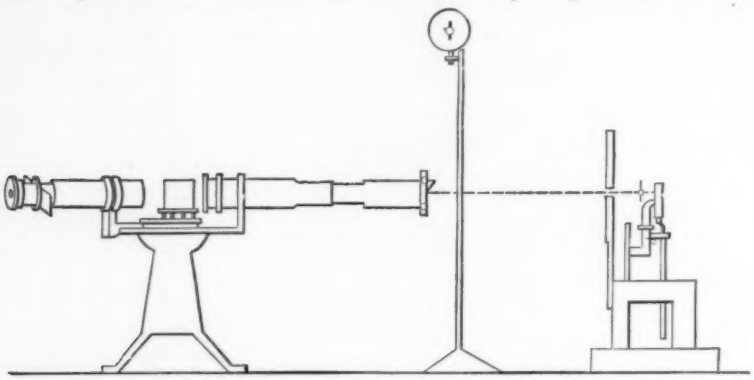

Fig. 4.

middle of the slit in order to secure a well-defined band of separation between the two portions of the field which were to be compared.

By means of this prism, rays from a glow-lamp placed about 30 cm. above the collimator tube were introduced into the upper half of the slit. This lamp was adjusted by previous trial to a degree of incandescence corresponding, as regards the distribution of energy in its spectrum, to gas light. For this purpose it was placed on a photometer bar, and compared with a bat's wing gas burner, the voltage of the lamp being adjusted until a photometer of a type very sensitive to color differences, showed no difference in the quality of the two lights. The voltage necessary to maintain the lamp in this condition was noted, and

throughout the subsequent experiments the lamp was maintained at the temperature corresponding thereto. In this way a steady source of comparison was obtained, the quality of the light from which was as nearly as possible identical with the light from a gas flame. While this is no very satisfactory basis for the definition of the quality of a light source, it appears to be the one least open to objection at the present day.

The eye-piece of the spectroscope was provided with a diaphragm by means of which similar regions in the two spectra to be compared could be isolated. The measurements consisted, as in all spectrophotometric work, in bringing the corresponding wave-lengths of the two spectra to equal brightness. This was accomplished in the present case by the adjustment to the Vierordt slit. In consideration of the fact that the source of light with which we had to deal changed rapidly with the time, the following method of taking readings was adopted: —

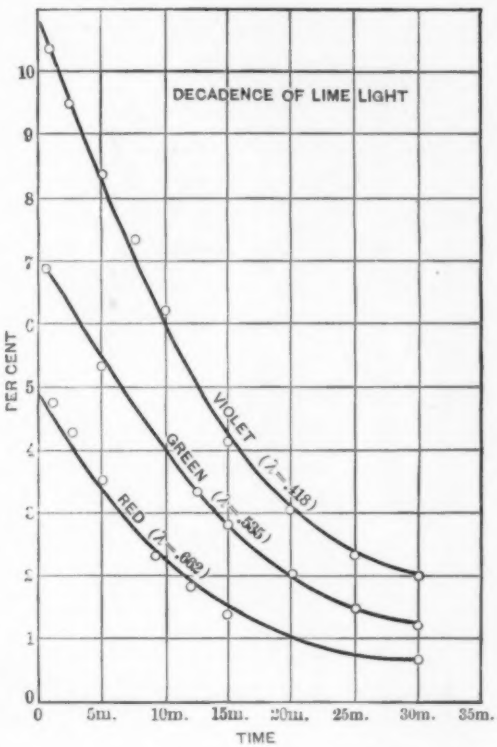

Fig. 5.

A fresh lime cylinder having been placed before the oxyhydrogen jet, the spectroscope was set to a chosen wave-length, the jet was ignited at a time carefully noted, and readings were taken at frequent intervals for this wave-length alone, until, after the lapse of about thirty minutes, it was found that the intensity of the

region under investigation had ceased to undergo further appreciable change of brightness. A new lime cylinder was then taken, the spectroscope was set to another wave-length, and the readings were repeated. In this way three regions, one in the red (0.662), one in the green (0.535). and one in the violet (0.418), were measured as functions of the time. The results are given graphically in Fig. 5. Finally, an incandescent lime cylinder which had been subjected to the action of a flame for half an hour, by which time it had come into a condition of nearly stable incandescence, was explored for relative brightness throughout its spectrum. For purposes of convenience, the region of the *D* line in this spectrum was taken as unity, and a curve was platted, the ordinates of which were the ratio $\frac{\text{lime light}}{\text{gas light}}$, this ratio being by definition equal to unity in the yellow.

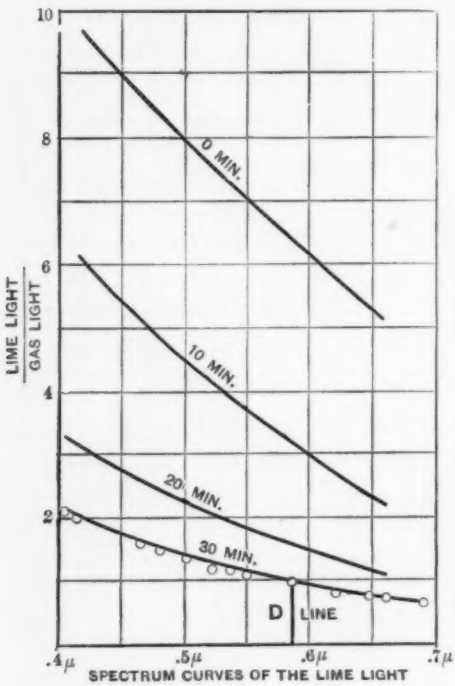

Fig. 6.

This 30-minute curve of relative brightness of the lime-light spectrum is given in Fig. 6. From this and the curves showing the decadence in the red, green, and violet as a function of the time (Fig. 5), it was possible to plat curves showing the character of the spectrum at any desired time after the ignition of the lime. Such curves are platted in Fig. 6 for the instant of ignition (0 min.), for 10 minutes and for 20 minutes.

In addition to these measurements some attempts were made to study the recuperative power of lime cylinders upon cooling. It was found that lime, after having been held in incandescence,

even for several hours, would, upon being allowed to cool, recover to a marked degree, acting upon re-ignition almost as though it were a fresh lime. This power of recuperation did not, however, appear to be uniform in the various specimens tested. It was much more complete in some than in others The time at the disposal of the experimenters did not admit of an exhaustive study of this interesting property.

The experiments described in this paper make it clear that lime possesses the remarkable property, long since known to exist in the case of other oxides, of luminescence by heat. In other words, its radiation when freshly ignited is one corresponding to a temperature very much higher than that to which it is actually subjected. That this is the case with magnesia, when the incandescent oxide is produced by the burning of magnesium ribbon, has been abundantly shown by Rogers in the paper already cited. One of the present writers in collaboration with Professor B. W. Snow[1] has shown that zinc oxide, even at a temperature of 1000° C., possesses, when first brought to incandescence, a radiating power vastly greater than that of platinum at the same temperature and superior to the latter, especially in the shorter wave-lengths. Also that this power is lost in the course of a few minutes. This property, which appears to be common to the many metallic oxides, affords a very interesting problem to the student of radiant energy. The present study is a contribution extending only to a single member of that group.

[1] Nichols and Snow, Philosophical Magazine [5], Vol. 32, p. 401.

A STUDY OF THE RESIDUAL CHARGES OF CONDENSERS AND THEIR DEPENDENCE UPON TEMPERATURE.

BY FREDERICK BEDELL AND CARL KINSLEY.

WHEN a condenser is subjected to a difference of potential for some time, part of the charge received is absorbed by the dielectric, and a part remains upon the surface of the condenser plates.[1] This latter part becomes discharged when the terminals of the condenser are connected together through a conducting circuit, and the condenser appears to be in a perfectly neutral condition. The absorbed charge, however, still remains in the condenser and will gradually come to the surface in the form of a residual charge of the same sign as before. If this be discharged, and the condenser be allowed to stand insulated, a second residual charge will collect in the same way as the first. A series of residual charges with rapidly diminishing values may thus be formed. Therefore, the condition of a condenser is dependent upon its past charges, some of which may have been held by the dielectric for weeks or even months before appearing upon the surface; a former charge of one sign may entirely neutralize a subsequent charge of opposite sign and smaller value. It has been justly said that the past history of a condenser must be known before it can be trusted.

The object of the present investigation was to determine this soaking-in effect of dielectrics. A study was made, first of the successive residual discharges of a neutral condenser and then of the effects produced by previously charging the condenser in the

[1] The part absorbed depends directly upon the duration of the time of charge; the part remaining on the surface is practically independent of the length of charge, for the condensers used in the present investigation. This was shown in a preliminary investigation by Messrs. W. M. Craft and H. B. Henderson, who showed the first discharge, as measured by the throw of a ballistic galvanometer, to be independent of the length of charge.

opposite direction. Figs. 1–10 show some results from this part of the investigation. The absorption by the dielectric was next studied by allowing a charged condenser to stand insulated and discharge through its own dielectric. The effect of a previous negative charge upon this absorption and upon the apparent resistance of the condenser was thus studied, as shown in Figs. 11 and 12. An investigation was then made of the influence of initial potential upon the discharge curves, and the insulation resistance, the results being shown in Fig. 13. An examination of temperature effects was then undertaken. The influence of temperature changes upon discharge curves through various commercial oils, and upon the resistance of the oils (see Figs. 14, 15, and 16), was ascertained preliminary to similar experiments upon condensers. Finally the influence of temperature upon the absorption by the dielectric of a condenser was obtained by a study of the successive residual discharges (Figs. 17 and 18), and by discharge curves (Fig. 19) at different temperatures. We will discuss separately the several parts of investigation.

The Effect of a Previous Negative Charge upon Successive Residual Discharges.

Previous to the final charge, the conductor was subjected, for this series of experiments, to a charge of opposite sign. The final charge was in every case in the same direction (arbitrarily called positive) and for the same length of time, viz. 30 seconds. Experiments were made with previous negative charges; *i.e.* charges opposite in direction to the final charge, of different duration. The value of the positive or negative potential to which the condenser was charged was in all cases about 114 volts. After the final positive charge of 30 seconds, the condenser was allowed to stand insulated for five seconds, and was then short-circuited for 15 seconds. After this discharge by short-circuiting, the condenser was allowed to stand insulated for definite intervals of time, during which the absorbed charges worked out to the surface. It was then discharged through a galvanometer, the throw of which furnished a comparative measure of the discharges.

Successive residual discharges were thus obtained by allowing the condenser to stand after each discharge for an interval of

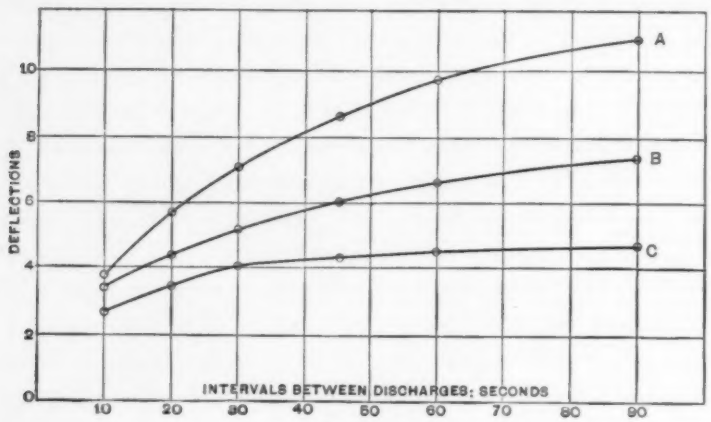

Fig. 1.

First Residual Discharge. Curve A for no previous charge: curve B for a previous negative charge for 30 seconds: curve C for a previous negative charge for 60 seconds.

time, during which the residual charges worked to the surface. The first, second, third, and fourth residual discharges were

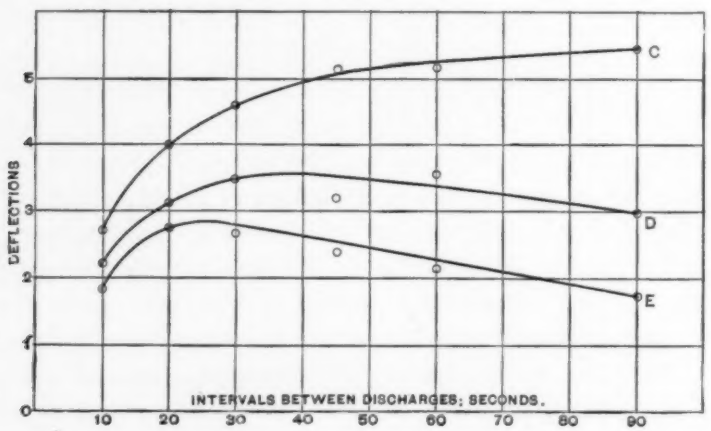

Fig. 2.

First Residual Discharge. Previous negative charge as follows: Curve C 60 seconds; curve D 90 seconds; curve E 120 seconds.

obtained with equal intervals of time between the successive discharges. This was repeated, allowing different intervals of

time between the discharges, and the results are shown in Figs. 1–8. In these figures, curves *A*, *B*, *C*, *D*, and *E* represent the residual discharges corresponding to different lengths of previous negative charge. In the case of curve *A*, there was no previous charge of any kind, the condenser being as nearly neutral as possible when the final positive charge of 30 seconds was given. The measurements for curve *B* were made upon the same condenser, which was given a previous negative charge for 30 seconds, was then allowed to stand insulated for five seconds, and then short-circuited for 15 seconds previous to the final posi-

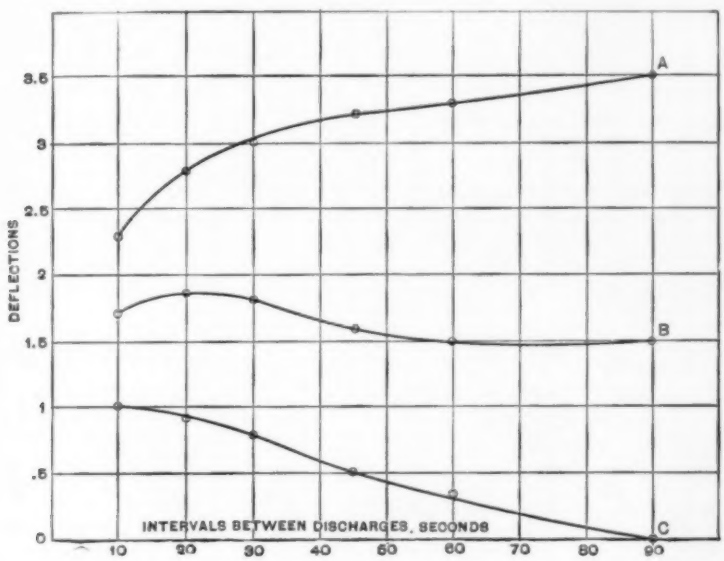

Fig. 3.

Second Residual Discharge.

tive charge. For curve *C*, the condenser received a preliminary negative charge for 60 seconds, stood insulated for five seconds, and was then short-circuited for 15 seconds previous to the final positive charge. For curves *D* and *E*, the previous negative charges were 90 and 120 seconds respectively, all other operations being as before. The absorption due to a positive charge deflected the needle to the right, and such deflections are plotted above the *X*-axis; deflections to the left, due to a negative charge,

are plotted below the axis. In taking curves *D* and *E*, the sensitiveness of the galvanometer was changed, and curve *C* was,

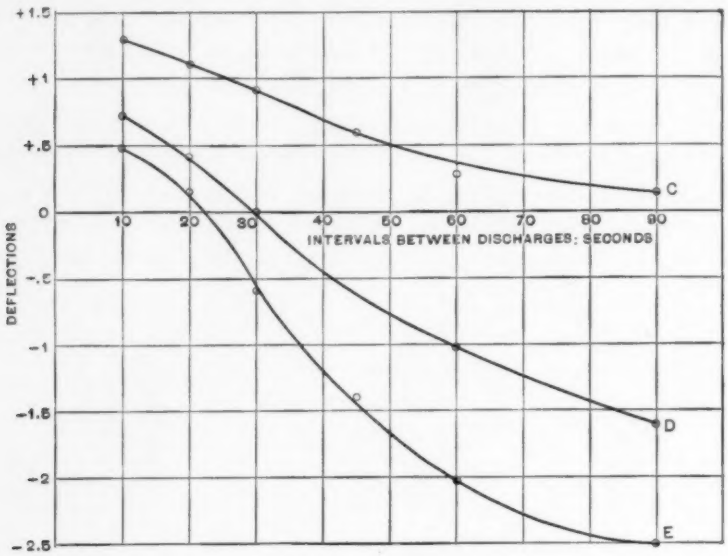

Fig. 4.

Second Residual Discharge.

therefore, determined twice, in order to show the relation between curves *A* and *B* and curves *D* and *E*.

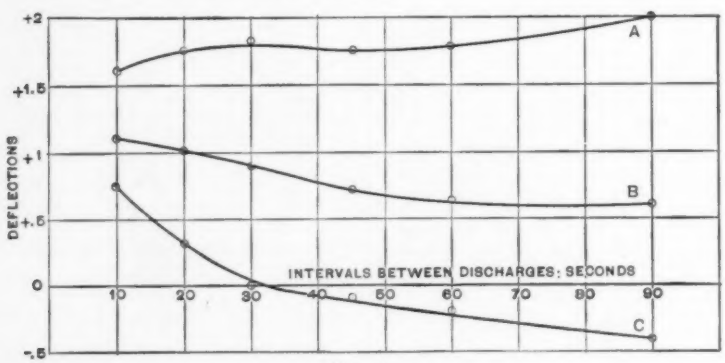

Fig. 5.

Third Residual Discharge.

The effect of the previous negative charge in partially or wholly neutralizing the final positive charge is clearly seen by an in-

spection of the curves. This influence becomes more and more marked with each successive discharge, and still more apparent as the intervals between the discharges are made greater. This would naturally be so, since the previous residual charge has time to work out and neutralize the absorption due to the final charge.

In the first residual discharge curves, Figs. 1 and 2, the previous charge has evidently the most influence where the intervals between discharges are the longest, and although the discharges are all positive, it is not at all unlikely that curves *D* and *E*, at

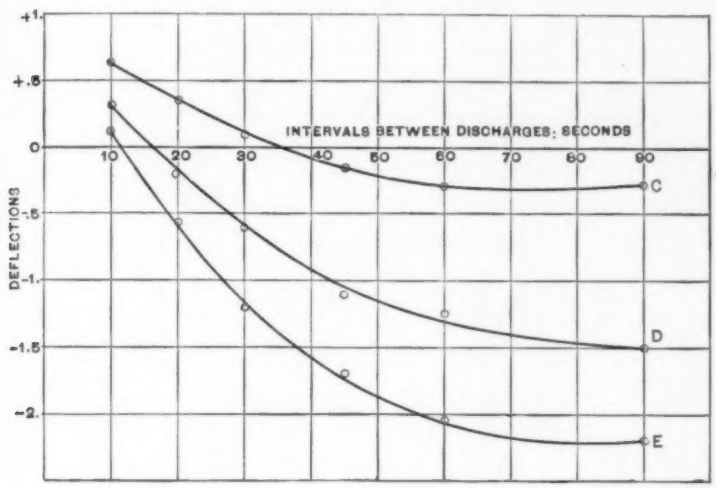

Fig. 6.

Third Residual Discharge.

any rate, would be brought to zero, or even made negative by increasing the intervals between discharges.

In the curves for the second residual discharges, Figs. 3 and 4, curve *C* is brought to zero, while curves *D* and *E* actually cross the *X*-axis, indicating a negative discharge; that is, one opposite in direction to the final charge. The evident turning again of the lower part of the curves indicates that the previous negative charge is becoming weakened.

All these effects are more marked in the curves for the third residual discharges, Figs. 5 and 6, where in the case of curves *D*

and *E*, the 10-second interval is the only one giving a residual discharge in the direction of the final charge. These curves are very nearly of the form which they would have if the final positive charge had not been given to the condenser at all, and the

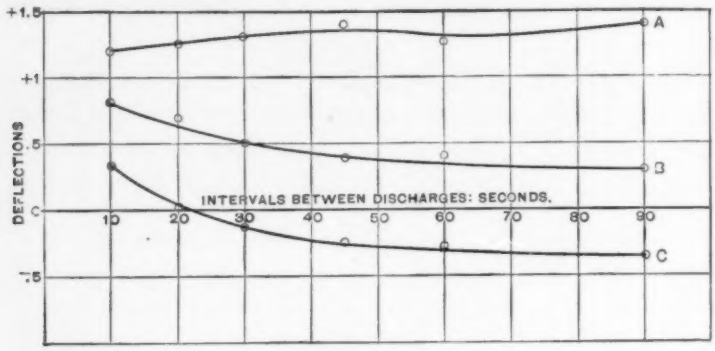

Fig. 7.

Fourth Residual Discharge.

residual discharges were due only to the previous negative charge, as is more plainly seen by inverting the figure. As is usual, the fourth residual discharge curves are rather flat, but curves *D* and *E*, as they indicate the presence of merely a negative

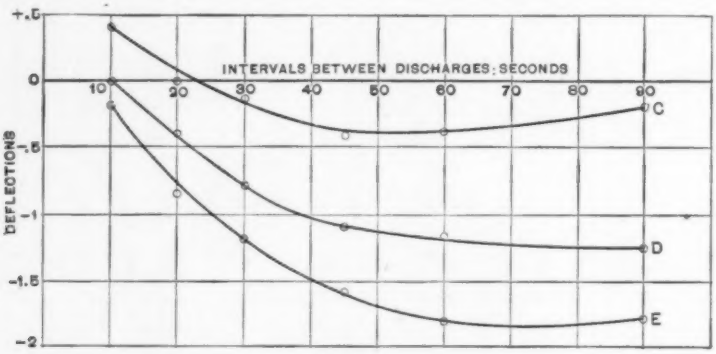

Fig. 8.

Fourth Residual Discharge.

charge, resemble the usual second discharge curves, and are considerably concaved towards the *X*-axis.

The fifth and sixth discharge curves were obtained, but were not sufficiently marked to warrant reproduction; they merely emphasized the foregoing observations.

The curves shown in Figs. 9 and 10 are obtained from the preceding curves, and show the effect of a change in the length of the previous negative charge given to the condenser before the final charging. The residual discharges, as measured by the throw

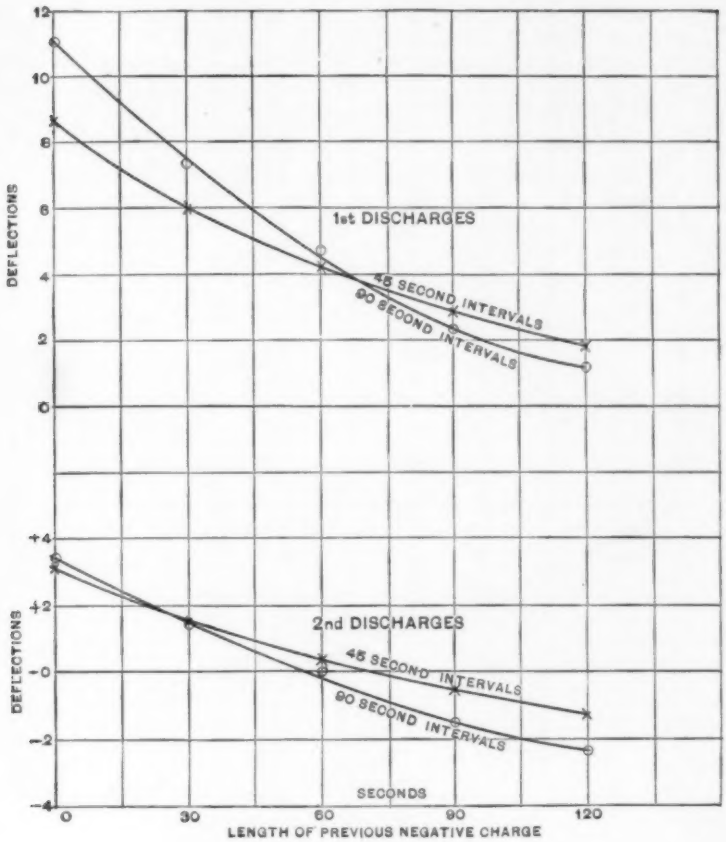

Fig. 9.

Effect of Previous Negative Charge upon Residual Discharges.

of the galvanometer, are represented as ordinates, and the lengths of the previous negative charges as abscissæ. Two sets of curves are drawn, obtained with intervals between the successive discharges of 45 and 90 seconds, respectively. In the case of the curves corresponding to intervals of 90 seconds, the residual discharges decrease with the increase of the previous negative

charge, more rapidly than for the corresponding curves taken with 45-second intervals between the successive discharges. This is in accordance with the interpretation of the previous results. These curves show plainly that the absorption of the previous

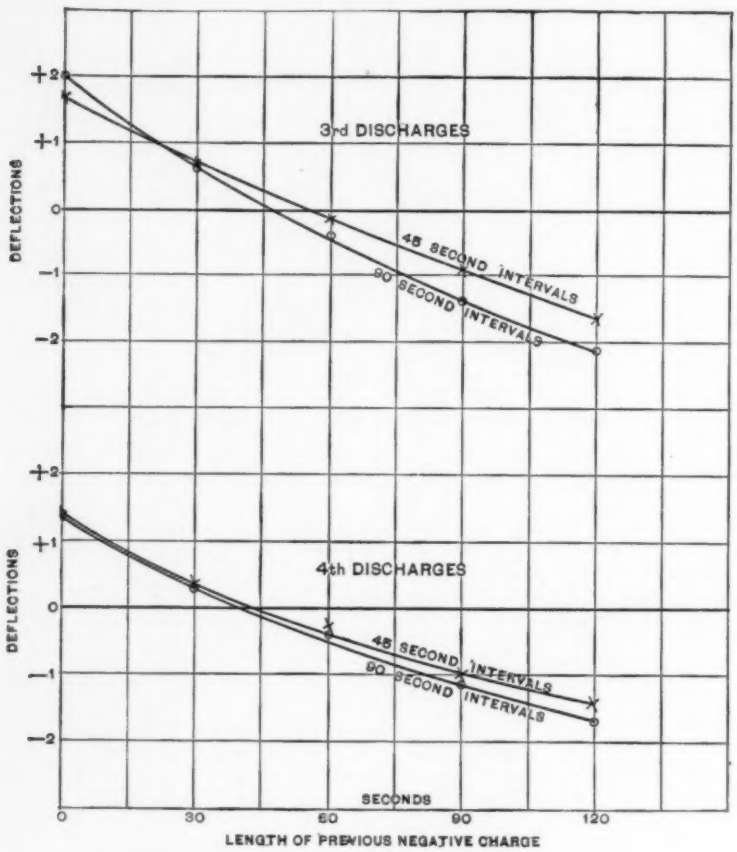

Fig. 10.

Effect of Previous Negative Charge upon Residual Discharges.

negative charge becomes greater as the time of this charge is increased; and the smoothness of the curves would indicate that the dependence of absorption upon time follows some definite law. The curves would probably be asymptotic to some line determined by the condition of the condenser and the nature of the dielectric. This line would then represent the limit to the amount of absorption.

The reflex nature of the curves, particularly of D and E, Figs. 4 and 6, suggested that they might be made to again cross the axis of X if three charges were given them, the first being in the direction of the final charge. This was done, and upon short-circuiting the condenser and then taking residual discharges at 10-second intervals, the galvanometer first showed positive, then negative, and finally again positive residual discharges.

With longer intervals between discharges it was found that residual charges of opposite sign more thoroughly neutralized each other. Hence, a condenser, if charged in both senses for different periods, would show before long a residual discharge merely in the direction of the greatest absorption. If the period were made long enough this would likewise leak away, and the curve would approach the axis of X; although, probably, the condenser would never become absolutely neutral.

The Effect of Absorption upon the Discharge Curves.

The curve for the discharge of a condenser through its own dielectric deviates widely from an exponential curve on account

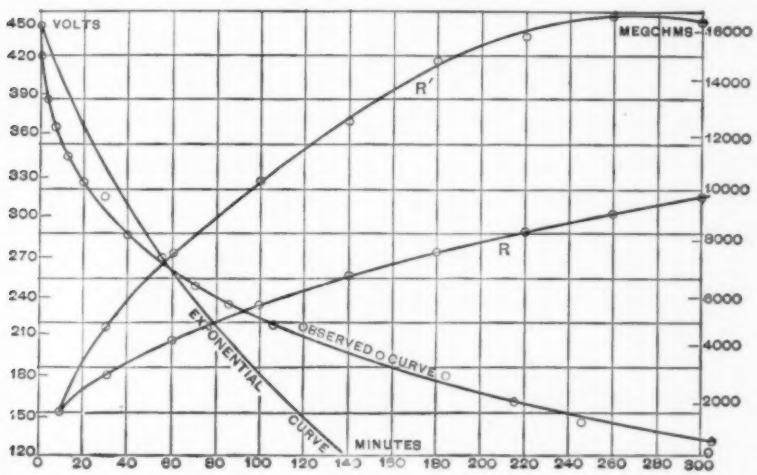

Fig. 11.

Condenser Discharge Curve and Computed Resistance. (Condenser No. 3.)

of the absorption of charge in the dielectric; for, as a condenser discharges, this residual charge works out to the surface and

becomes effective. The form of the discharge curve depends likewise upon previous charges, whether of the same or of opposite sign. The apparent resistance of a condenser is similarly dependent upon this absorption.

The discharge curves of the condenser were obtained by measuring the difference of potential at its terminals, by means of a multicellular voltmeter, as the condenser was allowed to discharge through its own dielectric. The leakage through the voltmeter

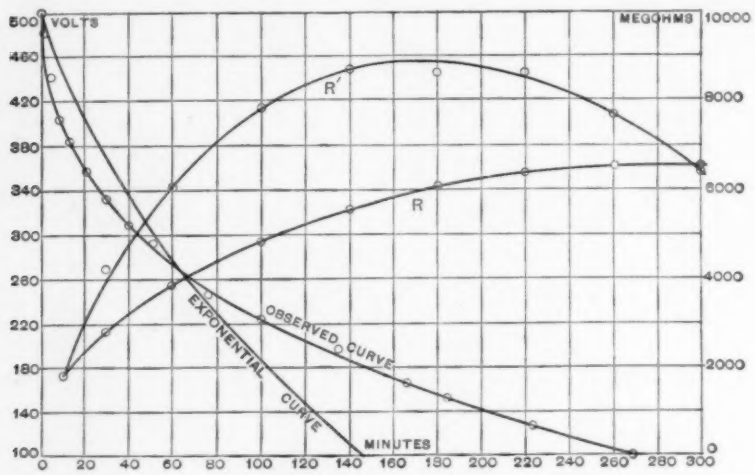

Fig. 12.

Condenser Discharge Curve and Computed Resistance. (Condenser No. 3.)

was found to be negligible. Assuming that the discharge of the condenser follows the exponential law, we have

$$V_t = V_0 \epsilon^{-\frac{t}{RC}},$$

where V_0 represents the initial potential; V_t the potential at some time t counted from the beginning of discharge; C the capacity of the condenser; R the insulation resistance, and ϵ the base of the Napierian logarithms. This gives

$$R = \frac{t}{C \log_\epsilon \frac{V_0}{V_t}}$$

This value of the resistance is the value which would be true if the discharge curve were exponential from the beginning of discharge to the time t.

The resistance may be otherwise computed by taking brief intervals of time and assuming the curve to be exponential for this time. Thus, if V_1 and V_2 are the potentials at the time t_1 and t_2, we have

$$R' = \frac{t_2 - t_1}{C \log_\epsilon \frac{V_1}{V_1}}$$

The value of the resistance thus obtained, which we will call R', is a closer approximation than the resistance R obtained above, and becomes more nearly a true value for the apparent resistance of a condenser as the interval between t_1 and t_2 is shortened.

In Fig. 11 is given the observed discharge curve from a condenser, which, although used for some time, had always been charged in the same direction. An exponential curve is also plotted with the assumed constant resistance of 4250 megohms. The derived curves showing the variation of the resistances R and R', obtained as explained above, are also given.

This condenser was left short-circuited for 88 hours, and then charged in the opposite direction, and the curve of discharge obtained as above. The results are shown in Fig. 12. The previous charge affects the shape of the discharge curves, but its influence is more clearly seen in the curves for the variation of resistance. The increase in apparent resistance, shown in Fig. 11, is due, as has already been explained, to the absorption in the dielectric. In Fig. 12 the resistance increases at first, due to the absorption of the final charge, and then decreases as the previous absorption of opposite sign begins to be effective. From this it is evident that the soaking-in effect, which is the cause of the usual rise of resistance, must have acted in the opposite direction, and so must have hastened instead of hindering the discharge. Where the curve for resistance R', calculated for short intervals of time from the discharge curve, is horizontal, the two absorbed charges neutralize each other, the resistance is constant, and the discharge curve is truly exponential.

The exponential curves, of course, cross the actual discharge curves when the resistance of the condenser is the same as the constant resistance assumed for the exponential curve. The resist-

ance assumed in the case of an initially neutral condenser should be so small that the exponential curve should lie entirely below the observed discharge curve, inasmuch as the apparent resistance is increased by the coming out of the charges previously absorbed.

The Influence of Initial Potential upon the Average Resistance.

To show more definitely that the departure of the discharge curve from an exponential form, and the variation of resistance

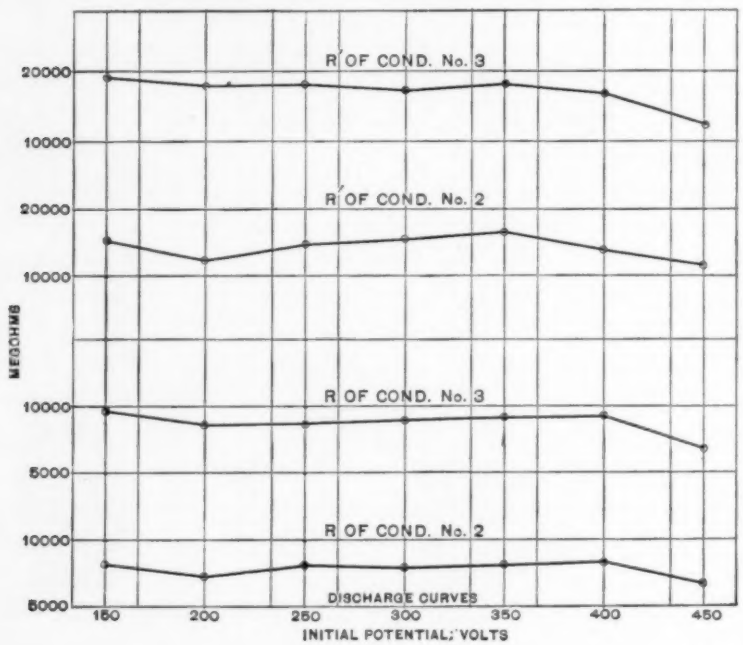

Fig. 13.

Average Resistance corresponding to Different Initial Potentials.

were due to absorption, and were consequently functions of the time rather than potential, discharge curves were taken through a wide range of initial potential, and the average resistances calculated by taking the mean values of the resistance obtained after 60, 160, and 260 minutes, according to the two methods above explained. Two condensers were used to check the accu-

racy of the work. The results agree throughout, and show a practically constant resistance throughout the range of initial potential employed; viz. from 150 to 450 volts, as shown in Fig. 13. The condensers were designed to stand a pressure of 500 volts, so we may conclude that while the insulation remains unstrained the changes which occur are due to what may be termed local causes — such as the conditions of the atmosphere and of previous charge — rather than to the initial potential of the condenser.

The Effect of Temperature on the Resistance of Oils.

The foregoing results show the effects of absorption in a solid dielectric. The results which follow show that the absorption in

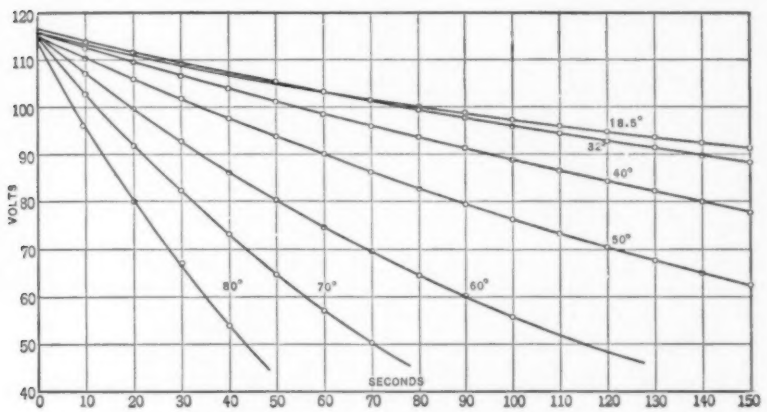

Fig. 14.

Discharge through Vacuum Oil at Different Temperatures.

ordinary dielectrics, as shown by the residual discharges and the curves of discharge of a condenser through the dielectric, are closely dependent upon temperature changes, and that at high temperatures the effects of absorption and residual discharge disappear entirely. An investigation of the effect of temperature changes upon the resistance of various commercial oils was made preliminary to a similar investigation upon solid dielectrics. The experiments for the determination of these temperature effects

were conducted, under the direction of one of us, by Mr. E. G. Gilson and Mr. E. G. Mason, and are described at length in their report[1] upon the work.

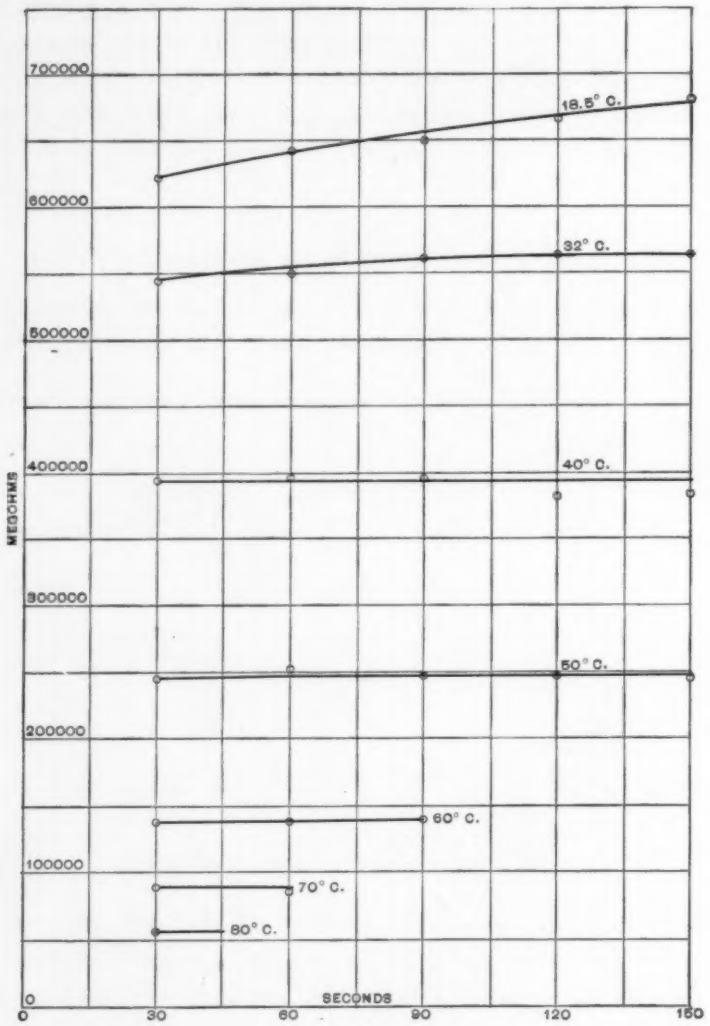

Fig. 15.

Resistance of Vacuum Oil at Different Temperatures.

[1] See MS. thesis in the Cornell University Library, "Effects of Temperature upon Dielectrics," by E. G. Gilson and E. G. Mason.

In perfect oils there should be no absorption of charge, and the curves for the discharge of a condenser through such oils should follow the exponential law. Two surface plates were immersed in the oil experimented upon; these plates were 25.25 cm. long by

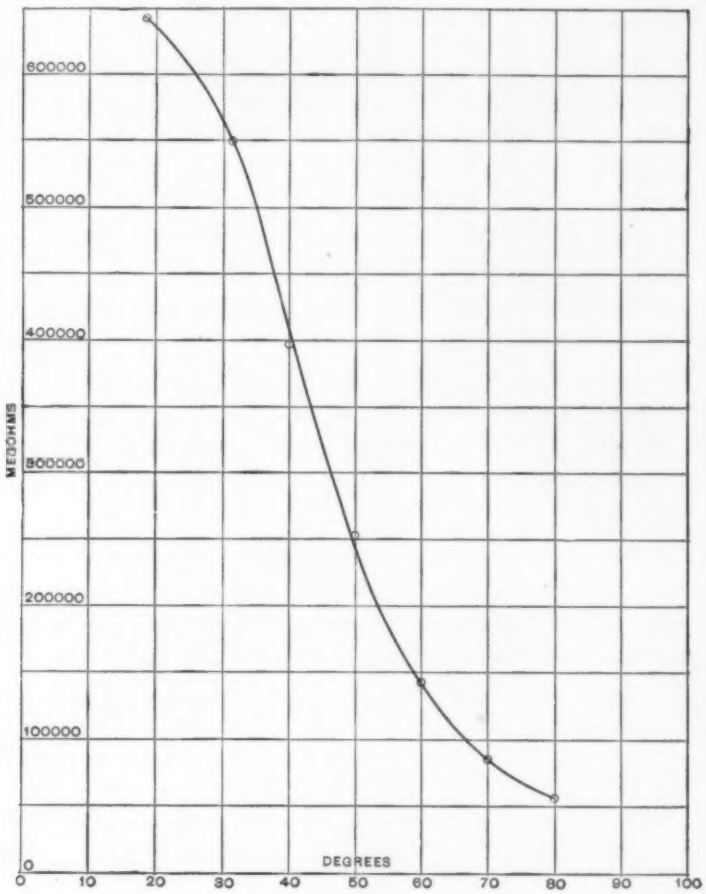

Fig. 16.

Resistance of Vacuum Oil at Different Temperatures after 60-second Electrification.

17.5 cm. wide, and were separated by three hard-rubber pillars 1.47 cm. high. The resistance of the oil was determined by discharging a condenser of known capacity through the oil by connecting the terminals of the condenser to these two surface plates.

The potential at the terminals of the condenser was measured by multicellular voltmeter, and the resistance computed according to the exponential law. The resistance of the rubber pillars, the multicellular voltmeter and of the condenser insulation were so small compared with that of the oil, that no correction for them was found necessary.

The results for different oils were similar, and curves are, therefore, given for vacuum oil alone.

Fig. 14 shows the curves for the discharge of a condenser through vacuum oil at different temperatures. The temperature of the condenser and its capacity were constant. The resistance of the oil computed from the discharge curves according to the first method already described, is plotted in Fig. 15. For the lower temperatures there is a slight increase in the resistance during the discharge. This has already been commented upon in the case of solid dielectrics, and indicates that there has been some absorption, possibly due to impurities in the oil. That the resistance is more nearly constant, and the discharge curves exponential at the high temperatures, indicates that absorptive effects disappear at these temperatures.

Fig. 16 shows the resistance of vacuum oil at different temperatures after an electrification[1] of 60 seconds. This decrease in resistance is most rapid between 30 degrees and 60 degrees where the resistance diminishes over 80 per cent. The same phenomena were observed in the experiments with boiled linseed oil, petroleum oil, and cylinder oil. The results are given in the accompanying tables.

[1] This word as here used may be misleading. It is used to denote the time the condenser stands electrified after being separated from the source of potential.

RESISTANCE IN MEGOHMS OF 1 CU. CM. OF VACUUM OIL.

Time in seconds.	18.5°	32°	40°	50°	60°	70°	80°
30	622,000	547,550	395,000	247,500	139,200	91,250	58,600
60	643,500	549,500	396,500	252,500	140,100	88,000	
90	649,900	560,000	396,600	249,500	142,000		
120	666,000	564,500	384,000	249,000			
130	682,000	564,000	383,500	245,000			

RESISTANCE IN MEGOHMS OF 1 CU. CM. OF CYLINDER OIL.

Time in seconds.	20°	40°	50°	60°	70°	80°	90°
30	141,100	136,000	78,200	60,300	40,850	36,830	37,420
60	141,000	140,950	78,750	63,200	42,700	38,750	39,300
90	162,950	142,500	80,350	64,200	53,600	39,550	40,450
120	165,800	146,900	82,100	63,950	44,220	40,300	40,500
150	185,200	148,200	83,600	64,900	44,550	40,850	41,650

RESISTANCE IN MEGOHMS OF 1 CU. CM. OF BOILED LINSEED OIL.

Time in seconds.	20°	30°	40°
10	—	1263	770
30	2310	1181	1084
60	2340		

RESISTANCE IN MEGOHMS OF 1 CU. CM. OF PETROLEUM OIL.

Time in seconds.	13°	20°	30°	40°	50°
30	44,700	38,900	28,150	20,200	14,200
60	45,650	39,990	28,210	19,740	
90	46,550	40,850	28,720		
120	47,150	40,200			
150	47,550				

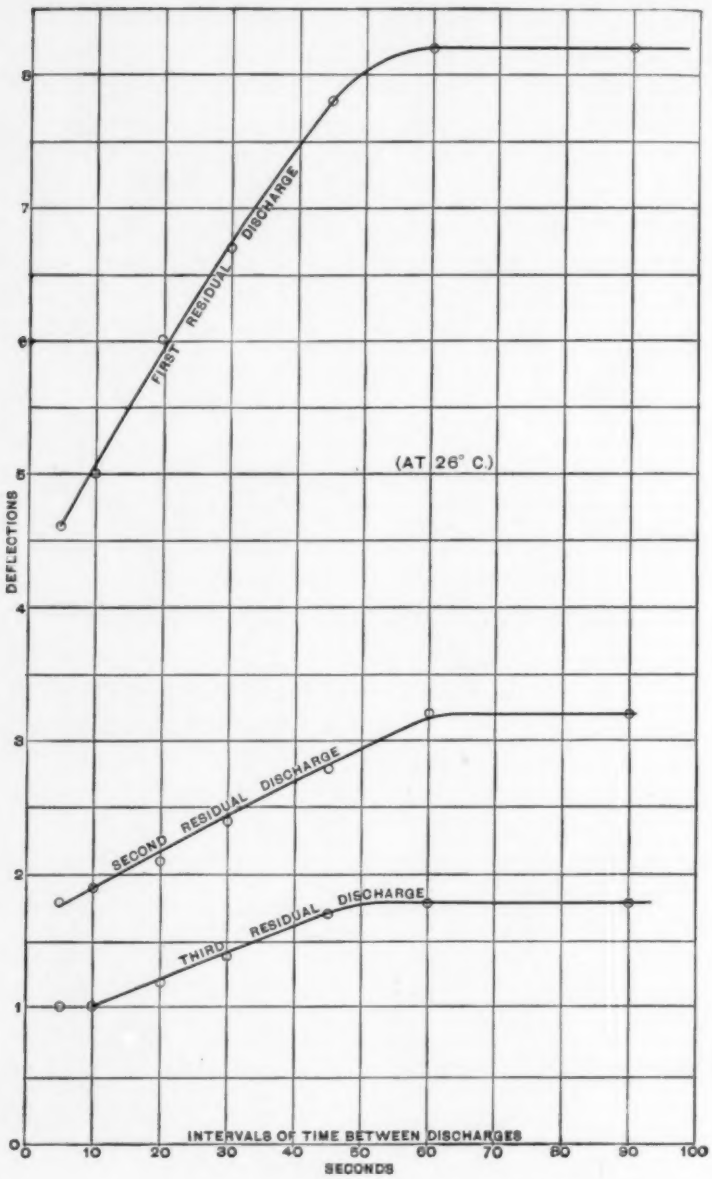

Fig. 17.

Residual Discharges of Condenser at 26° C.

Temperature Effects upon Solid Dielectrics.

For this investigation the same condensers were used as in the experiments described in the earlier parts of this paper. An investigation of the dielectric hysteresis of these condensers and the effects of temperature upon capacity have already been described at length in a paper[1] in the PHYSICAL REVIEW, where a detailed description of the condensers is also given.

A determination of the capacity of the condenser was made in connection with these earlier experiments, and also during the present investigation a year later. The values are here given for comparison.

Condenser.	No. 2.	No. 3.	No. 4.
Capacity (Microfarads) in 1893	1.571	1.52	1.539
" " " 1894	1.592	1.535	1.554

The measurements were made in each case by the method of mixtures, but with different apparatus, so that some of the variation may be due to the change of standards. The condensers, however, had stood severe treatment, being in continual use for a year under all sorts of conditions, and a slight change of capacity might be looked for.

Temperature Effects upon Residual Discharges.

The first, second, and third residual discharges are obtained at different temperatures by the method used in the experiments already described. The sensitiveness of the galvanometer was not kept constant, so that the deflections cannot be compared so as to show the relative values of the absolute quantity of electricity constituting the residual discharges under different conditions. The shape of the curves plotted in Figs. 17 and 18, however,

[1] Bedell, Ballantyne, and Williamson, "Alternate-Current Condensers and Dielectric Hysteresis," PHYSICAL REVIEW, Vol. i., No. 2.

indicates the relative amount of absorption at different temperatures, which is the point in question.

The residual discharges at 26° C. show a large amount of absorption, as it took long intervals between discharges to equalize the residual discharges.

Residual discharges at 32° C. give curves much flatter; the

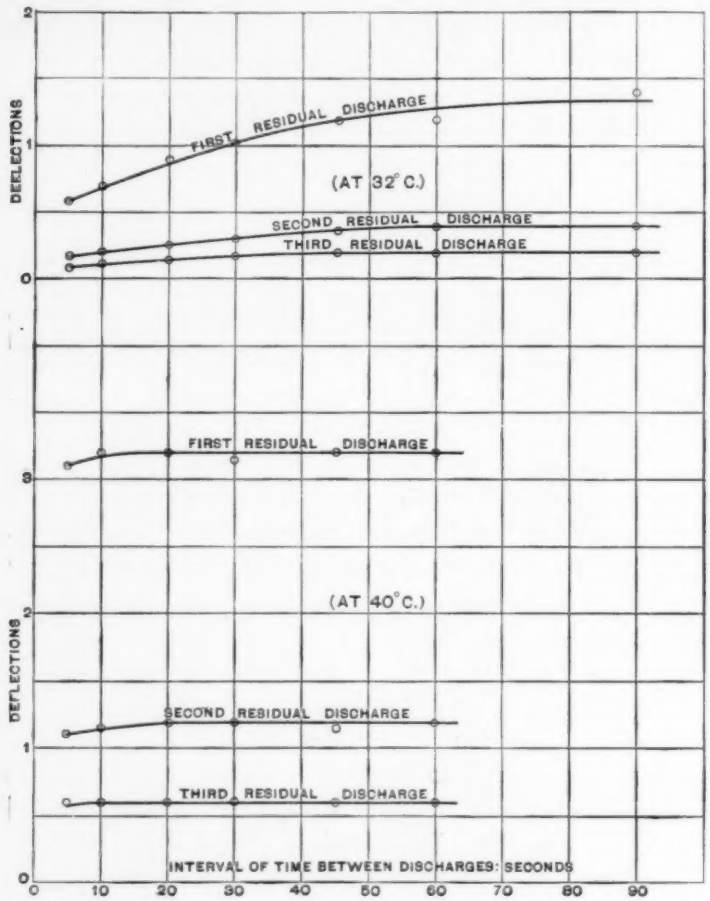

Fig. 18.

Residual Discharges of Condenser at 32° C. and 40° C.

interval between discharges does not largely affect their amount. Residual discharges at 40° C. give remarkably flat curves, the intervals between discharges having, practically, no effect upon

the discharges. This indicates that there was but little absorption. Whatever residual charge there was, so readily passed out that a large increase in the intervals between discharges did not affect the quantity of the residual discharge.

These experiments show conclusively that absorption, as shown by residual effects, disappears at high temperatures.

Temperature Effects upon Discharge Curves and Insulation Resistance.

The insulation resistance of a condenser was obtained by allowing the condenser to discharge through its own insulation, and

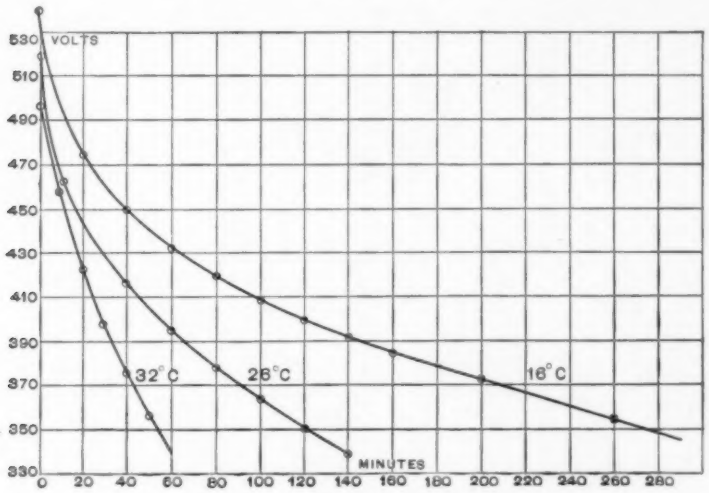

Fig. 19.

Condenser Discharge Curves at Different Temperatures.

then solving the usual exponential equation for the resistance. The capacity of the condenser at different temperatures was known as the result of the experiments already referred to.

The discharge curves at different temperatures are given in Fig. 19; the differences are due, however, to change in both capacity and resistance.

The variation in resistance during the discharge, computed according to the first method before described, is shown in Fig. 20, and is to be compared with Fig. 15, which shows the cor-

responding results for vacuum oil. Fig. 20 shows that the resistance remains more nearly constant at high temperatures. Inasmuch as this variation of resistance during discharge is due to absorption, it is evident that there is less and less absorption

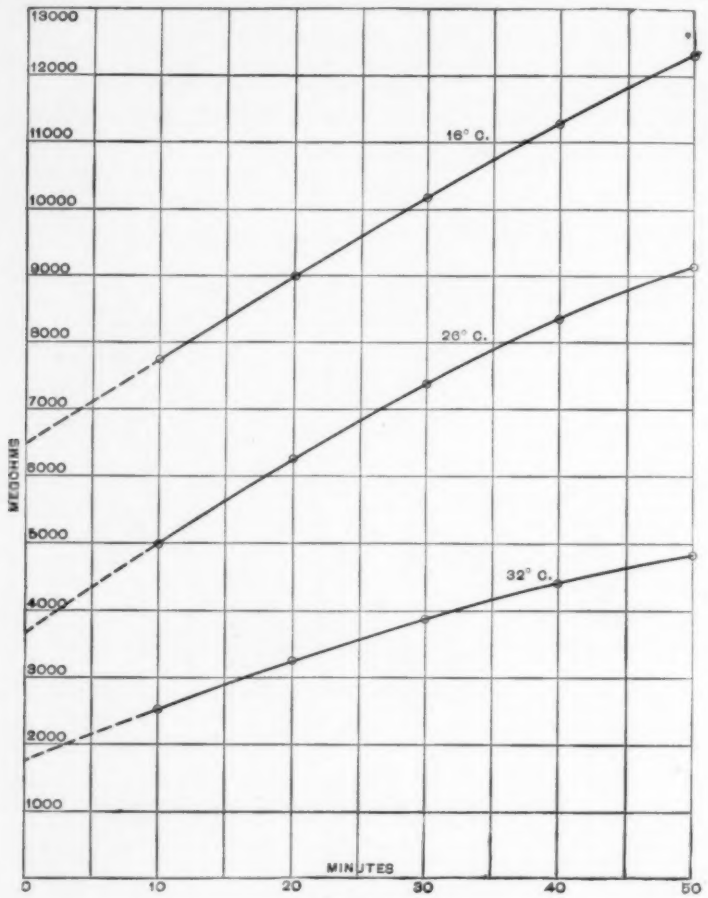

Fig. 20.

Condenser Resistance after Different Times of Electrification at Different Temperatures.

as the temperature rises, a conclusion already reached from a study of residual discharges.

The condenser resistance at different temperatures, after three minutes of electrification,[1] is given in Fig. 21, and is in form quite

[1] See preceding note.

similar to the corresponding curve for vacuum oil (Fig. 16). The sudden bend in a curve after 40 degrees corresponds to the rapid changes in capacity at these temperatures. Between 42 degrees

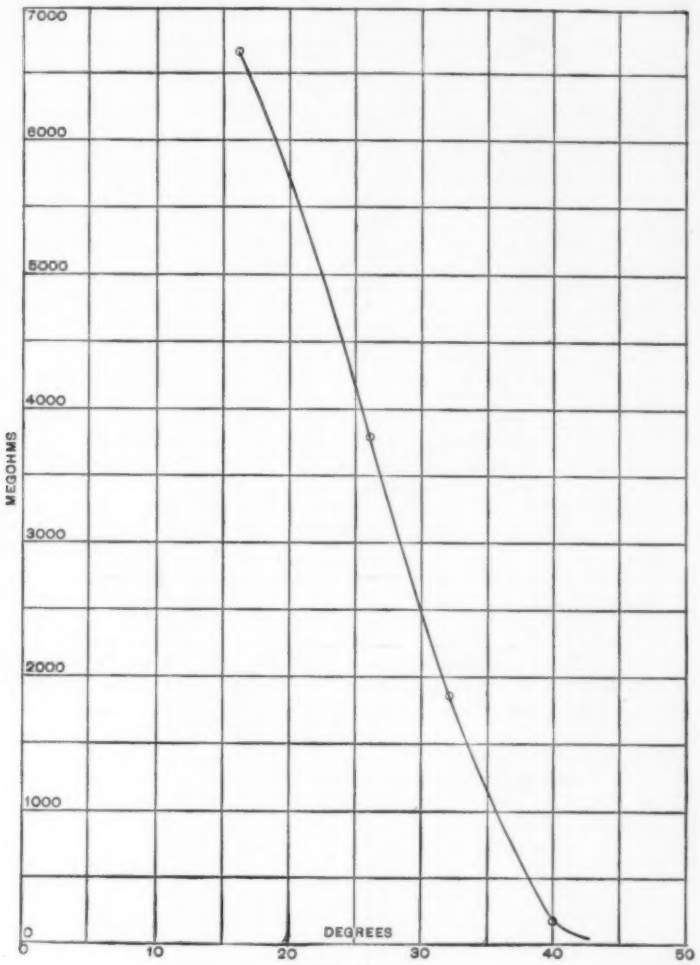

Fig. 21.

Condenser Resistance for Different Temperatures after Three Minutes' Electrification.

and 50 degrees the capacity increases very rapidly. When the capacity becomes infinite, the resistance will, of course, be zero.

Mr. M. A. Hess read a paper before La Société Française de Physique, a review of which appeared in *L'Électricien*, January,

1894, in which he says, in speaking of the phenomena of absorption: "If the product of the capacity and resistance of different portions of the dielectric sheet are equal, there will be no residual discharges. . . . It has been observed that the insulation dimin-

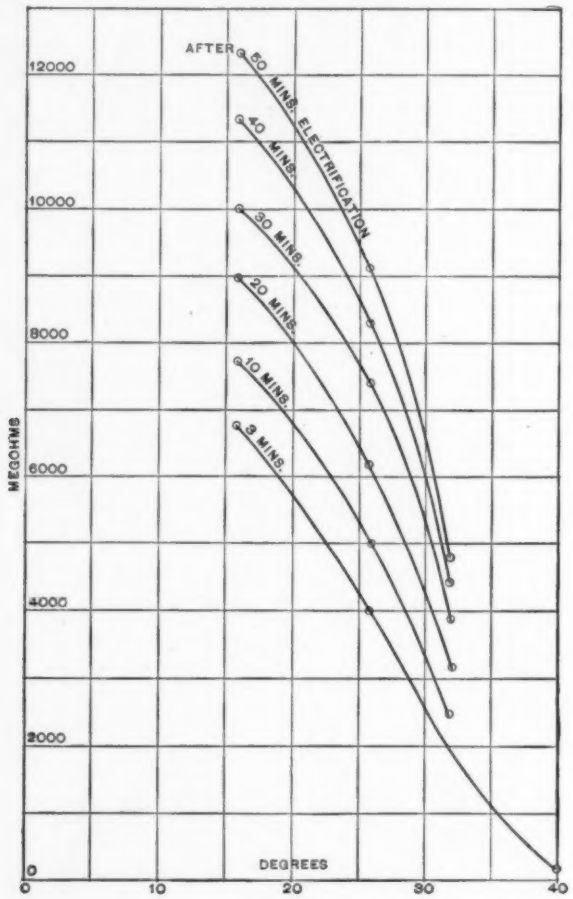

Fig. 22.

Condenser Resistance for Different Temperatures after Different Times of Electrification.

ishes when the temperature rises. . . . From this it is seen that there is a temperature at which the phenomena of the residual charge disappears."

In Fig. 22 curves are drawn after different times of electrifica-

tion similar to the curve in Fig. 21. These emphasize the results already discussed. These curves come nearer and nearer together at high temperatures as absorption is eliminated, and apparently would coincide at some temperature between 40° C. and 50° C., the limiting temperature the condenser could stand.

The experiments given above would indicate that in the case of the paraffin condenser, absorption will never be eliminated until the capacity becomes infinite. The temperature was raised until the resistance became only one-twentieth of its normal value. The absorption, though diminishing, was very definite in amount, and the condenser would evidently break down before the residual discharges entirely cease.

Conclusion.

The action of a condenser is dependent, in a marked degree, upon its previous history; so much so, in fact, that its previous charges may be of more importance in determining its action under certain circumstances than charges received later. In that case the previous absorption, after neutralizing the absorption due to the final charge, may give rise to residual discharges which increase with time up to their previous initial value.

Absorption gives rise to the phenomena of residual charges, and causes the condenser to depart from the exponential law in discharging through its own dielectric. The increase in apparent resistance in the condenser during discharge is associated with this effect of absorption. Previous charges modify these results, increasing or decreasing them according to whether the previous charges were in the same or opposite sense.

The resistance of pure oils is constant at any one temperature, but falls off rapidly with the temperature. There are, accordingly, no absorption or residual effects in pure oils.

In solid dielectrics the effects of absorption are diminished as the temperature is increased, as shown by the residual discharges and by the changes in the insulation resistance.

This investigation can in no wise be considered as comprehensive. Many of the results here described are already known,

but, as far as the writers are aware, they have not before been presented together so as to show the relation between them. The fact that the previous condition of the condenser has such an influence upon its action, causes considerable embarrassment in an investigation of this sort, inasmuch as a condenser used once under certain conditions may be practically useless in the same investigation for further experiments. One of the condensers experimented upon retained its past charges after being short-circuited for a month, and it was necessary to employ another condenser, which was neutral, for further experiments.

A GENERAL THEORY OF THE GLOW-LAMP. II.[1]

By H. S. Weber.

III.

THE definitive quantity in the case of the glow-lamp is the energy which the electric current develops in the filament in a unit of time. By means of this the temperature of the filament is determined when the surface is known, and thus finally the quantity of light which the filament radiates is given. One of the fundamental questions in the domain of glow-lamp illumination is that of the relation between the amount of light radiated and the expenditure of energy necessary to its production. Voit has attempted to find a simple expression for this quantity based upon experiments with incandescent lamps made at the Munich electrical exhibition. As has already been pointed out he found that the light radiated was neither proportional to the square nor to the fourth power, but approximately proportional to the cube of the energy consumed. "The formula $H=qE^3$ may be regarded as an empirical rule which for glow-lamps is between about 6 c.p. and 120 c.p." For the constant q, Voit found, from the Munich measurements:—

Edison lamp	(16 c.p.)	$q = 37.6 \times 10^{-6}$	Siemens lamp	(16 c.p.)	$q = 22.5 \times 10^{-6}$
Edison lamp	(8 c.p.)	$= 110.6 \times 10^{-6}$	Small Müller lamp	(20 c.p.)	$= 21.3 \times 10^{-6}$
Small Swan lamp	(10 c.p.)	$= 84.8 \times 10^{-6}$	Medium Müller lamp	(50 c.p.)	$= 6.7 \times 10^{-6}$
Large Swan lamp	(40 c.p.)	$= 9.6$	Large Müller lamp	(100 c.p.)	$= 2.1 \times 10^{-6}$
Maxim lamp	(28 c.p.)	$= 14.8$	Cruto lamp	(10 c.p.)	$= 25.0 \times 10^{-6}$

The constant q according to these measurements is very different in the case of different lamps, and as Voit has pointed out, it is small in proportion as the useful light energy of the lamp increases. No further investigation concerning the factors which

[1] A paper read at the Frankfort Congress for Electrotechnics. Translated for the PHYSICAL REVIEW by E. L. N.

are involved in this coefficient was attempted by Voit, and nothing has since been done in this direction. In my measurements of thirty-three types of glow-lamps, I have found for every lamp Voit's empirical relation, according to which the intensity of light is proportional to the cube of the energy consumed, fairly well established. It appeared, however, that the size of Voit's coefficient is not quite a constant, but that it increases as the light increases from small intensities, then more rapidly with increasing brightness to a maximum, and finally diminishes again.

A glance at the column marked $q=\frac{H}{E^3}$ of the last four tables shows the change in this quantity with increasing HE as just described. At the same time it appeared from measurements, as Voit had shown, that the size of the coefficient for different lamps is enormously different. As a mean value of q we find from the above tables:—

For the Woodhouse lamp	41.0×10^{-6} and 32.0×10^{-6}
For the Sunbeam lamp	2.12×10^{-6} and 1.49×10^{-6}
For the lamp of the Allgemeinen Electricitäts-Gesellschaft	87.9×10^{-6} and 70.4×10^{-6}
For the Cruto lamp	75.7×10^{-6} and 55.8×10^{-6}

where H is the mean horizontal intensity or the mean spherical intensity respectively. It is easily seen that all these phases are represented in a formula for radiation already presented. The intensity of any given visible homogeneous ray of wave-length λ is given through the formula:—

$$s = I\frac{c}{\lambda^2}\pi F e^{aT-\frac{1}{b^2\lambda^2T^2}}.$$

The brightness of this radiation is proportional to that quantity. In order to obtain an expression for the total brightness, we may take advantage of the fact that the candle-power of a solid glowing body increases in proportion to the brightness of the homogeneous ray, of which the wave-length is about 0.54μ. If we set in this last equation in place of λ this value, then the expression gives us the relation between the total brightness and the temperature.

The energy which is consumed in the glowing filament when

the same has been warmed to a stationary temperature T is, according to the above equation,

$$E = ICFTe^{aT}.$$

If we take cognizance of the fact that the constant of total radiation has a value of

$$\tfrac{1}{2}\pi\sqrt{\pi}bc,$$

we obtain as an expression for the quotient H/E^3 the following formula: —

$$\frac{H}{E^3} = \frac{m}{C^2F^2} \cdot \frac{1}{T^3 \cdot e^{2aT + \frac{1}{b^2\lambda^2T^2}}} = \frac{m}{C^2F^2}\phi(T), \qquad (6)$$

in which small m is a constant. In order to determine the function $\phi(T)$, we must know, aside from the quantities $a = 0.0043$ and $\lambda = 0.54$, the constant b^2 for carbon. From Langley's observations concerning the distribution of energy in the spectrum of black carbon at a temperature of 178°, I find the quantity $b^2 = 0.192 \times 10^{-6}$, where the wave-lengths were measured in microns. From my own measurements upon the change in brightness of homogeneous rays from the gray carbons of the Sunbeam lamp, I determined for the temperature interval $T = 1450° \cdots T = 1650°$, $b^2 = 0.190 \times 10^{-6}$. If we assume for b^2 the latter value, then the quantity $b^2\lambda^2$ is equal to 0.0052×10^{-6}.

In general, the function depends in a complicated way upon the temperature. If, however, we have to consider only the temperature interval from 1400° to 1600°, within which range the temperature of the glow-lamp lies, it may be shown that the character of this function for the interval in question is very simple. The following table shows the change of $\phi(T)$ for successive intervals of 10°: —

T	$\lg \Phi(T)$	$\Phi(T)$	T	$\lg \Phi(T)$	$\Phi(T)$
1440°	−42,959	0.953 × A	1520°	−42,913	0.998 × A
1450	−42,945	0.966	1530	−42,916	0.995
1460	−42,934	0.977	1540	−42,920	0.991
1470	−42,924	0.987	1550	−42,927	0.984
1480	−42,918	0.993	1560	−42,936	0.975
1490	−42,914	0.997	1570	−42,947	0.965
1500	−42,912	0.999	1580	−42,959	0.953
1510	−42,911	1.000			

The last vertical column of this table indicates that the function $\phi(T)$ between $T=1400°$ and $T=1580°$ goes through precisely the same changes as the factor q already described. It increases slowly from 1400° on, reaches a maximum of 1510°, the value of which is expressed by the equation

$$3+2aT-\frac{2}{b^2\lambda^2T^2}=0,$$

and then diminishes slowly. Between 1475° and 1542° the value of $\phi(T)$ changes only 1 per cent; between 1463° and 1555°, 2 per cent; from 1554° to 1565°, 3 per cent.

Observations with the various individual lamps are in entire accord with this result; viz. the maximum for q in the case of the Woodhouse lamp occurs in the neighborhood of 1415°; for the Sunbeam lamp, in the neighborhood of 1498°; for the Berlin lamp, 1503°; and for the new Cruto lamp, finally, at 1514°.

We may safely assume, then, that if the value of the function $\phi(T)$ for a temperature of 1510° is A, the quantity $q=HE^3$, for the entire range of temperature between 1452° and 1465°, will change at most 3 per cent from the constant value mAC^2F^2. Further, since the maximum value of q is

$$q_{max}=\frac{mA}{C^2F^2},$$

the value of $q_{max}C^2F^2$ must be constant for all lamps. This relationship is found to be true in fact, as the following tables for the four lamps will show:—

	q_{max}	F^2	C^2	$q_{max}C^2F^2$
Woodhouse lamp .	33.10×10^{-6}	0.790 cm.^{4}	0.285×10^{-9}	7.3×10^{-15}
Sunbeam lamp . .	1.57	25.400	0.172	6.9
Berlin lamp . . .	72.30	0.577	0.167	7.3
New Cruto lamp. .	56.50	0.399	0.292	6.6

The small variation for the value given in the last vertical column of this table is easily accounted for, since these quantities involve the square surface of the filament, which is difficult to determine, and the square of the constant of total radiation.

The mean value of this quantity ($q_{max}C^2F^2$) I found for all the carbons investigated to be 6.9×10^{-15}.

Since q changes only about 1 per cent between 1460° and 1560°, we may assume with a high degree of approximation for this interval, for all lamps, the relation

$$H=\frac{6.8\times 10^{-15}}{C^2F^2}\cdot E^6;$$

where H is the mean spherical candle-power expressed in British candles.

IV.

In its application to the glow-lamp, the relationship between the energy consumed in the filament in a unit of time, and the resulting brightness of radiation, is the most important quantity. Ever since the first measurements upon lamps were made it has been known that with increasing temperature of the filament this quantity diminishes very rapidly. But the law of the relationship between this quantity and the temperature has not been accurately known. The formulas which we have already developed from our observations enable us to express this relation in a simple manner.

Since the energy expended in a unit of time is

$$E=ICFTe^{aT}, \text{ where } C=\tfrac{1}{2}\pi\sqrt{\pi}\,cb,$$

and since the brightness expressed in any units — for example, British candles — is given by the expression

$$H=a\frac{c}{\lambda^2}\cdot\pi Fe^{aT-\frac{1}{b^2\lambda^2T^2}},$$

in which a is a factor of proportionality, and λ is the value 0.54, it follows that the quantity $E_1=\frac{E}{H}$ is given by the equation

$$E_1=\frac{I\pi\lambda\sqrt{\pi}}{2a}\cdot b\lambda T.e^{\frac{1}{b^2\lambda^2T^2}}. \qquad (7)$$

The energy necessary to develop a unit of light in a unit of time is consequently proportional to the temperature of the function, and depends upon the value of

$$\Psi(b\lambda T)=b\lambda Te^{\frac{1}{b^2\lambda^2T^2}}.$$

The change of this temperature of the function for the given value of $b^2\lambda^2=0.0552\times10^{-6}$ for the interval of temperature applicable to the glow-lamp, 1400° to 1650°, is given in the following tables for intervals of ten degrees. At the same time values of this function for the higher temperatures, 1700°, 1800°, 1900°, and 2000°, are given, in order to indicate the further progress of this function at higher temperatures.

T	$\Psi(b\lambda T)$	T	$\Psi(b\lambda T)$	T	$\Psi(b\lambda T)$
1400	3501	1500	1135	1600	455
1410	3093	1510	1027	1610	419
1420	2740	1520	931	1620	387
1430	2433	1530	846	1630	358
1440	2166	1540	770	1640	332
1450	1934	1550	702	1650	308
1460	1731	1560	642	1700	214
1470	1552	1570	588	1800	115
1480	1395	1580	539	1900	68
1490	1257	1590	495	2000	44

On account of the great importance of the quantity E, an experimental proof that E is in fact such a function of the temperature, as it is represented to be by Equation 7, is of considerable value. For the purposes of this proof we may make use of the contents of the tables which have already been given to illustrate the properties of the four different glow-lamps. If we seek by means of these tables the values at the temperatures indicated, and divide these values by the value of $\Psi(b\lambda T)$, for the same temperatures we should obtain a constant quotient in case E really is of the form given in Equation 7.

The numbers given in the last columns of the following four tables are these quotients. It will be seen that for each of the four lamps the fluctuations are such as to size and direction that they may well be ascribed to errors of observation.

LAMP OF THE BERLIN CO. (ALLGEMEINE ELEK.-GESELLSCHAFT).

E_1	T	$\Psi(\delta\lambda T)$	$\frac{\Psi(\delta\lambda T)}{E_1}$
10.87 W.	1464°	1659	153
8.67	1483	1354	156
7.07	1503	1103	156
5.88	1522	914	155
4.91	1541	763	155
4.18	1557	640	158
3.57	1574	568	158
3.09	1591	491	158
2.71	1607	430	158
2.41	1621	384	159

SUNBEAM LAMP.

E_1	T	$\Psi(\delta\lambda T)$	$\frac{\Psi(\delta\lambda T)}{E_1}$
9.64 W.	1463°	1677	174
8.26	1478	1426	173
6.70	1498	1159	173
5.20	1524	897	173
4.11	1549	709	172
3.27	1573	573	176
2.68	1595	475	177
2.25	1615	403	179
1.94	1636	342	176
1.69	1654	299	177

NEW CRUTO LAMP.

E_1	T	$\Psi(\delta\lambda T)$	$\frac{\Psi(\delta\lambda T)}{E_1}$
14.03 W.	1434°	2326	166
10.63	1460	1731	163
8.41	1482	1367	163
6.08	1514	989	163
5.09	1532	831	163
4.32	1550	702	163
3.70	1566	610	165
3.14	1582	530	169
2.66	1600	455	171
2.31	1618	393	170

LAMP OF WOODHOUSE AND RAWSON.

E_1	T	$\Psi(b\lambda T)$	$\frac{\Psi(b\lambda T)}{E_1}$
20.60 W.	1400°	3501	175
12.37	1441	2141	173
8.20	1479	1411	172
5.77	1514	989	172
4.21	1548	716	170
3.63	1561	637	175
3.17	1578	549	173
2.76	1592	487	176
2.55	1606	433	170
2.23	1620	387	173

The contents of these four tables can also be used to demonstrate that the four different filaments to which they apply have the same or nearly the same values for the constant b. If b^2 is the same for these four carbons and equal to 19×10^{-6}, then the quotients must be the same for all four lamps. In order to compare these quotients among themselves, however, it is first necessary to take cognizance of the question of spherical intensity, since these lamps have very different relations between the mean spherical and the mean horizontal candle-power, as is shown by the following tables: —

	$\Psi(b\lambda T) : E_1$	R	$R\Psi(b\lambda T) : E_1$
Berlin lamp	157	0.801	126
Sunbeam lamp	175	0.711	124
New Cruto lamp	165	0.734	121
Woodhouse lamp	173	0.780	135

If we apply to each lamp its proper reduction factor, we find that the correct value of the quotient given in the last column is nearly the same for the four types. It follows from the equation for E_1, already given, that

$$-\frac{dE_1}{E_1} = \frac{db}{b} \cdot \left(\frac{2}{b^2 \lambda^2 T^2} - 1 \right),$$

a result which shows that the percentage of decrease in E_1, which is caused by a given percentage of diminution in b, is as much larger than the factor

$$\left(\frac{2}{b^2\lambda^2T^2}-1\right)$$

as the latter is larger than 1. This factor, however, has the following values (when $b^2\lambda^2=0.0552\times10^{-6}$); viz.

17.48 when $T=1400°$	14.08 when $T=1550°$
16.23 when $T=1450°$	13.15 when $T=1600°$
15.10 when $T=1500°$	12.31 when $T=1650°$

The change in the constant b of n per cent of its value would also produce a change in the value of E_1, which change would be from 12 to 17 times as great.

The tables given can also be made to furnish an answer to the question of the percentage of decrease in E_1 when the temperature rises one degree.

From Equation 7 it follows that

$$-\frac{dE_1}{E_1}=\frac{dT}{T}\left(\frac{2}{b^2\lambda^2T^2}\right)-1,$$

from which we can calculate the diminution of E_1 for one degree rise of temperature. The result is as follows:—

$\frac{1}{17}\times 17.48\,\% = 1.24\,\%$ for 1400°	$\frac{1}{15.5}\times 14.08\,\% = 0.91\,\%$ for 1550°
$\frac{1}{14.5}\times 16.23 = 1.12$ for 1450°	$\frac{1}{17}\times 13.15 = 0.82\,\%$ for 1600°
$\frac{1}{17}\times 15.10 = 1.01$ for 1500°	$\frac{1}{16.5}\times 12.31 = 0.75\,\%$ for 1650°

Within the interval of temperature of 1400° to 1650° a rise in temperature of one degree produces, very nearly, a diminution of one per cent in the amount of energy necessary for the production of a unit of light. For this interval of temperature, therefore, a rise of n degrees in temperature will improve the economy of light production by nearly n per cent.

V.

The result of the discussion in Section III., according to which the total brightness of the glow-lamp may be expressed by means of the constant, $a=6.8\times10^{-15}$, the square of the radiating surface,

the square of the constant C, the total radiation, and the cube of the expenditure of energy, gives us the means of determining the dimensions which a carbon filament must have in order to give a certain intensity of light under prescribed conditions. In order to calculate the length and the radius of the filament, which we will assume is of circular cross-section, we must know the difference of potential at which the lamp is to be used, the candle-power H which is to be produced, the temperature at which this candle-power is to be delivered, and finally the nature of the carbon, with required specific resistance, and the constant C of total radiation. Since the consumption of energy per unit of total brightness is determined by the temperature, we can set in place of the temperature a given value for E_1. From these six quantities α, ΔP, H, E_1, w, and C, the dimensions of l and ρ may be determined in the following manner: —

$$H=\frac{\alpha}{C^2F^2}\cdot E^3\text{; or since } E=E_1\cdot H,$$

$$H=\frac{\alpha}{C^2F^2}E_1^3H^3 \text{ and } F_2=\frac{\alpha}{C^2}E_1^3H^2,$$

or
$$l^2 4\pi^2\rho^2=\frac{\alpha}{C^2}\cdot E_1^3H^2.$$

The resistance of the carbon is

$$\frac{lw}{\pi\rho^2}=\frac{\Delta P^2}{E}=\frac{\Delta P^2}{E_1H}.$$

From these two equations it follows that

$$l=\sqrt[3]{\frac{\alpha H\cdot E_1^2\cdot\Delta P^2}{4\pi wC^2}};$$

$$\rho=\sqrt[6]{\frac{\alpha w^2\cdot H^4\cdot E_1^5}{4\pi^4\cdot C^2\cdot\Delta P^4}}$$

If the cross-section of the filament be rectangular, with one side a and the other b, and if $a=bn$, where b is a given number,

the equations for the determinations of b and a are as follows: —

$$l=\sqrt[3]{\frac{\alpha H E_1^3 \cdot \Delta P^2}{\frac{4}{n}(1+n)^2 w C^2}};$$

$$a=\sqrt[6]{\frac{\alpha w^2 \cdot H^4 E_1^b}{4(n+n^2)^2 \cdot C^2 \Delta P^4}}.$$

In making use of these operations it is to be noted that the assumed value of the constant α, 6.8×10^{-15}, applies to mean spherical candle-power, and that the specific resistance of the carbon is to be taken as the resistance per cubic centimeter, the dimensions of the filament being expressed in centimeters.

As an example of the usefulness of this formula the following illustration may serve: A Siemens lamp, the carbon of which had a specific resistance $w=0.00209$ at its normal temperature, and a radiation coefficient $C=0.0000129$, gave 16 mean horizontal candles when the potential difference was 98 volts. The efficiency was 3.4 watts per candle. The reduction factor of this lamp to mean spherical candle-power was 0.71. In this case, then, H was equal to 11.36, and E_1 to 4.79. The measured length of the filament was found to be 15.58 centimeters, and the radius, measured at nine points equally distant along the thread, was 0.00756 cm. According to the formulas just given for l and ρ, l should be 15.74 cm., and ρ should be 0.0077 cm.

The measured dimensions were somewhat smaller than the calculated, which may result from the fact that the measurements were made at ordinary temperatures.

VI.

The general expression for radiation affords an example of the formula for the optical efficiency of the glow-lamp (the relation between the visible energy of radiation and the total radiation). The total energy of the radiation is

$$S'=\int_{\lambda_1}^{\lambda_2} \frac{c}{\lambda^2} \cdot \pi F \cdot e^{aT-\frac{1}{\beta\lambda^2 T^2}} \cdot d\lambda,$$

where λ_1 and λ_2 are the wave-lengths of the borders of the visible spectrum. A further development of this form gives: —

$$S' = c\pi FbTe^{aT}\left\{\int_{\frac{1}{b\lambda_2 T}}^{\infty} e^{-x^2}\cdot dx - \int_{\frac{1}{b\lambda_1 T}}^{\infty} e^{-x^2}dx\right\}.$$

If we take as wave-length boundaries of the visible spectrum the values 0.38 μ and 0.78 μ; and if we set $b^2 = 0.19 \times 10^{-6}$, and assume as our interval of temperature 1400° to 1650°, the lower limit of the second integral will fall between 4.07 and 3.46, and as a consequence the value of the second integral for all these values will be smaller than 10^{-7}. S' may then be reduced to the following form: —

$$S' = c\pi FbTe^{aT}\int_{\frac{1}{b\lambda_2 T}}^{\infty} e^{-x^2}dx.$$

The energy of the total radiation, according to Equation 3, is

$$S = c\pi FbT\frac{\sqrt{\pi}}{2}\cdot e^{aT},$$

so that the expression for the optical efficiency is

$$N = \frac{S_1}{S} = \frac{2}{\sqrt{\pi}}\cdot\int_{\frac{1}{b\lambda_2 T}}^{\infty} e^{-x^2}dx.$$

For $T = 1500°$ the value of this integral is 0.0049, and $N = 0.55\,\%$.
For $T = 1550°$ the value of this integral is 0.0065, and $N = 0.73\,\%$.
For $T = 1600°$ the value of this integral is 0.0083, and $N = 0.94\,\%$.
For $T = 1650°$ the value of this integral is 0.0104, and $N = 1.17\,\%$.

For normal temperatures of glow-lamps, therefore, the efficiency is in the neighborhood of 1 per cent.

An experimental determination of this kind gives from 4 per cent to 5 per cent. (Measurements made by Blattner under my direction in 1885 gave about 5 per cent. Those made later by Merritt gave about 4 per cent.) These, however, were dependent upon the assumption that an aqueous solution of alum absorbed every part of the dark heat and let through all the visible rays. Since, however, this is not quite true, and these liquids let through

some of the dark rays which lie near the boundary of the visible spectrum, the values experimentally obtained must be too large. If these solutions, for example, absorbed only those dark rays the wave-lengths of which are greater than (1.0 μ), the calculated value for the efficiency would be

$$N = \frac{2}{\sqrt{\pi}} \cdot \int_{\frac{1}{bT}}^{\infty} e^{-x^2} \cdot dx,$$

and the numerical values of the efficiency as follows: —

$T = 1500°$; $N = 3.0\,\%$.
$T = 1550°$; $N = 3.7\,\%$.
$T = 1600°$; $N = 4.3\,\%$.
$T = 1650°$; $N = 4.9\,\%$.

VII.

It has long been known that the rapid diminution in the life of glow-lamps with rising economy is to be explained in the fact that carbon at a certain temperature begins to suffer notable vaporization, which with further rise of temperature increases in intensity. After having found it possible to determine the temperature of carbon filaments for every condition of incandescence, it seemed to me important to fix the temperature at which carbon begins to show evaporation, and to show whether this temperature is the same for all sorts of filaments. The investigations which I have made in this direction are based upon the observations of the resistance of filaments. The resistance of the carbon filament remains constant at a given temperature as long as its cross-section is maintained. When the temperature is reached, however, at which appreciable quantities of carbon evaporate from the surface of the filament, then the resistance suffers a slow increase with time. If we raise the temperature of the filament, maintaining it constant at each step for a considerable time, and notice during this time the change of resistance, we may obtain an estimate of the temperature at which a considerable increase of resistance occurs.

As time intervals, during which the chosen temperatures were to be held constant, I selected thirty hours. By means of accu-

mulators of large capacity, it is not at all difficult to regulate the strength of the current, so that the energy developed in the filament will remain constant to within one-thousandth of its value. From time to time during the test the current strength and the difference of potential were read, and the resistance was calculated from these values. The observations were thus continued until a temperature was found at which a sensible steady increase of resistance with the time became apparent. As an example of these measurements the following list of observations may be taken. They refer to the Woodhouse lamp in which $F=0.895$ cm.^{2}, $R=0.780$, and $C=0.0000169$.

Series I.	Series II.	Series III.
$\imath =$ 1.4458 amp.	1.5107 amp.	1.5615 amp.
$\Delta P =$ 60.52 v.	62.80 v.	64.74 v.
$W =$ 41.86 ohms	41.57 ohms	41.46 ohms
$E =$ 87.50 w.	94.87 w.	101.09 w.
$H =$ 28.01 c.p.	35.03 c.p.	42.26 c.p.
$E_1 =$ 3.12 w.	2.70 w.	2.39 w.
$T =$ 1580°	1596°	1610°

The resistances were as follows : —

Hours.	Ohms.	Ohms.	Ohms.
0	41.86	41.57	41.76
3	41.86	41.59	41.47
7	41.87	41.56	41.49
10	41.85	41.58	41.52
20	41.86	41.60	41.57
26	41.85	41.59	41.59
30	41.87	41.59	41.62

In a similar manner a lamp of the Berlin Company was tested, in 1888 at 1590°. It was found that the resistance of this lamp remained constant for thirty hours. At 1603°, on the contrary, it increased decidedly, rising to larger and larger values. It appears, therefore, that the temperatures at which various specimens of carbon begin to vaporize are somewhat different.

MINOR CONTRIBUTIONS.

On the Thermal Conductivity of Cast Iron as a Function of Temperature.

By I. Thornton Osmond.

IT is well known, from the experiments of Forbes and of Tait, that the thermal conductivity of wrought iron diminishes rapidly as the temperature increases, until the temperature of about 150° C. is reached, after which the conductivity changes less rapidly and becomes a minimum at about red heat.

Having reason to believe that this was not true for cast iron, and being unable to find any investigation on the subject, I had Messrs. Jenkins and Lingle, of Bellefonte, Pa., cast some bars and pieces from the No. 1 foundry pig iron made at the Bellefonte furnaces from Centre Co., Pa., ores: a soft gray cast iron, low in carbon, and high in silicon. Mr. George W. Kessler, a candidate for the master's degree, undertook the investigation, and presented the details of the work and tables of the observations very fully in a thesis, from which the observational data of this article are drawn; that is, Tables I., II., and III.

The method employed was the classic one used by Forbes for wrought iron; but to avoid cutting large cavities for thermometers, temperatures were measured thermo-electrically.

The principal apparatus, and its arrangement in use, are sufficiently indicated in Fig. 1. The galvanometer used was of the Thomson, astatic, mirror pattern; coils 5.84 ohms.

The experimental work embraced four principal operations, viz.: —

(1) To determine the size, density, and specific heat of the bar of cast iron.

(2) To calibrate the set of apparatus.

(3) To find the temperatures along the bar.

(4) To find the rate of cooling of a short piece of the same cast iron as the bar, and same diameter.

1. *The Bar.*

From fragments of pieces cast from the same iron, at the same time that the bar was cast, the density was found to be 7.140, and the specific heat 0.1155.

The bar principally used was 66 cm. long, with a projection at right angles at one end 6 cm. long, and was turned to 3.28 cm. diameter.

On the upper side of this bar, at distances 6, 8, 10, 12, 14, 16, 19, 22,

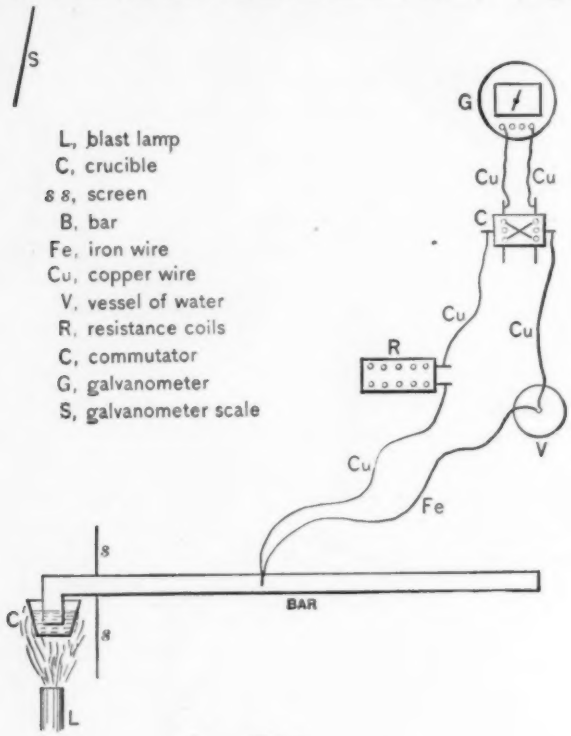

Fig. 1.

25, 29, 33, 37, 42, 47, 52, 58, and 64 cm. from the furnace end, holes 3 mm. diameter were drilled half-way through, for the insertion of the thermo-couple. These holes, the points of observation along the bar, are numbered from 1 to 17 in the order of their distances given above. See abscissæ of Curve II.

2. *Calibration of Apparatus.*

After some preliminary trials with the apparatus arranged as in Fig. 1, by which the resistance to be used, position of the controlling magnet of the

galvanometer, etc., were determined, the junction of the thermo-couple was removed from the bar and put in a vessel of mercury at over 200° C., the cool junction being in a vessel of water at 21° C. The thermometers used were without errors that need be considered in this work, shown by recent testing. As the mercury cooled, the observations given in Table I. were made by simultaneous readings of temperature and deflection.

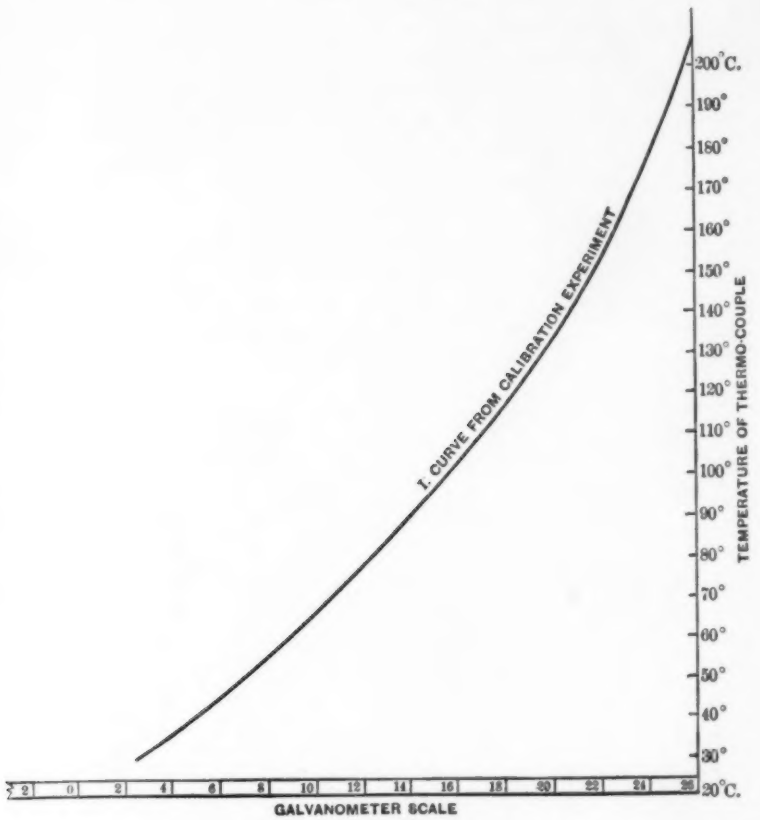

Fig. 2.

This calibration is shown in Fig. 2. Observations giving a curve almost indistinguishable from one here shown were taken at another time, serving both to test the accuracy of this set, and to show the unchanged condition of the thermo-couple.

TABLE I. (See *Curve I.*)

Temperatures.		Deflections.
Cool junction.	Hot junction.	Scale readings.
21° C.	200° C.	25.7
21° C.	176° C.	24.2
21° C.	153° C.	22.2
21° C.	131° C.	19.7
21° C.	110° C.	16.7
21° C.	93° C.	13.7
21° C.	77° C.	11.2
21° C.	64° C.	8.7
21° C.	51° C.	6.2
21° C.	40° C.	3.9

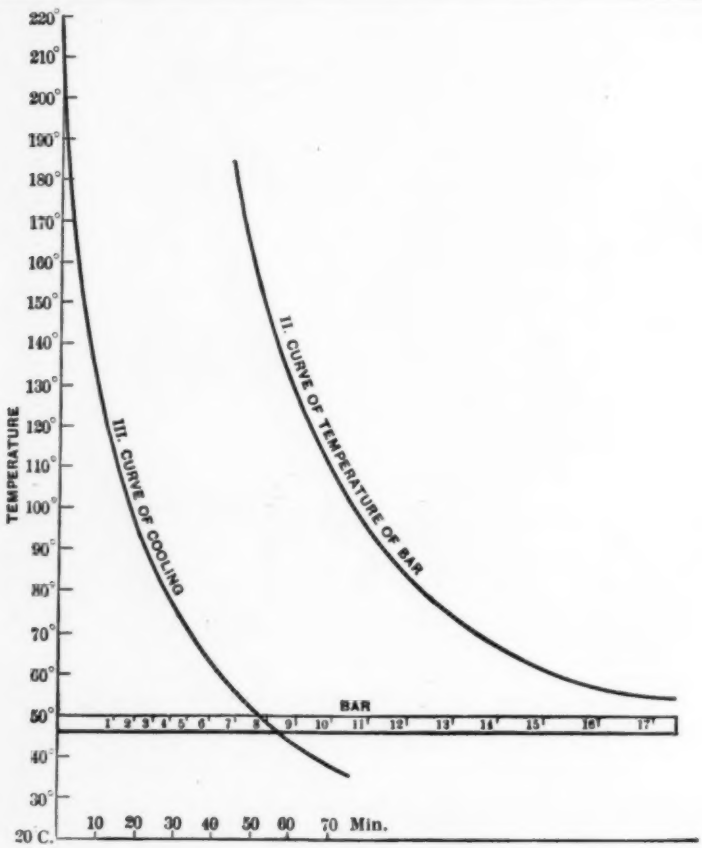

Fig. 3.

3. *Temperatures along the Bar.*

The right-angled projection at one end of the bar was put in a small crucible (set in asbestos and brick) in which by a strong and very steady blast lamp aluminum was melted. To insure good thermal contact, mercury was placed in the holes along the bar; but finding that it vaporized from some of them (during the long heating necessary), careful trial showed that its presence or absence gave no observable effect.

After a number of preliminary experiments, to get everything in order and to ascertain the best adjustment, the bar was heated, on May 25, for many hours, until the deflection obtained from any one point ceased to vary, showing a steady condition of temperature at each point.

Four sets of observations were then made along the bar, as given in Table II. The cool junction of the thermo-couple was in a vessel of water at 21° C., and there were 14 ohms unplugged in the resistance box, as in the calibraton experiment. The other junction was inserted successively in several holes along the bar, and the deflections read as soon as they were steady. These results are shown in curve II., Fig. 3.

TABLE II. (See *Curve II.*)

Point of bar.	Obs. I. Scale reading.	Obs. II. Scale reading.	Obs. III. Scale reading.	Obs. IV.[1] Scale reading.	Mean.
1	25.0	24.1	—	—	6.65
2	26.4	26.1	24.25	24.3	7.26
3	26.0	27.3	26.05	26.4	8.39
4	25.4	25.7	25.45	—	9.51
5	26.0	26.6	26.3	26.2	11.50
6	26.8	26.2	26.7	—	13.92
7	25.7	25.1	25.95	26.8	16.00
8	23.4	23.2	23.75	—	18.65
9	21.1	21.5	22.05	22.3	21.74
10	18.4	18.7	18.85	—	23.45
11	15.9	16.15	16.35	15.6	25.64
12	13.6	14.1	14.05	—	26.50
13	11.3	11.55	11.55	11.6	26.27
14	9.1	9.8	9.85	9.3	25.71
15	8.0	8.4	8.45	8.7	26.44
16	6.9	7.35	7.2	7.6	25.26
17	5.9	6.9	6.9	6.9	24.55

[1] The exhaustion of the blast pressure made it necessary to go rapidly through part of Obs. IV., omitting some points.

4. *Curve of Cooling.*

A separate short bar of the same iron, cast at the same time as the principal bar and turned to the same diameter, after a long heating to a high temperature, had one junction of the same thermo-couple previously used with the main bar inserted in the same manner as in the latter: the same apparatus and conditions being in circuit.

The observations made while this piece cooled are given in Table III., and shown by curve III. in Fig. 3.

TABLE III. (See *Curve III.*)

Minutes . .	0	1	2	3	4	5	6	7	8	9
Deflection .	26.7	26.1	25.4	24.7	24.0	23.3	22.6	21.9	21.2	20.6
Minutes . .	10	11	12	13	14	15	16	17	18	20
Deflection .	20.0	19.4	18.7	18.2	17.6	17.0	16.6	16.1	15.5	14.6
Minutes . .	22	25	30	35	40	50	60	70		
Deflection .	13.7	12.4	10.6	9.2	8.1	6.0	4.5	3.5		

The inclinations of the tangents to curve III. represent rates of cooling per minute in degrees centigrade.

All these curves lie so closely upon the points platted that in the reduced drawing herewith the points, except two or three of curve II., are inseparable from the curves.

5. *Results. Table IV.*

In Table II. it will be noticed that the deflections at the first six points of observation are irregular and anomalous. On converting the deflections into temperatures, by use of curve I., the reason of this is evident. Point 6 of the bar is nearly at the temperature of inversion of the copper-iron thermo-couple. Hence, the following *Table of Results* begins with the 18th centimeter of the bar, lying between points 6 and 7. Column 2 is obtained from curve II., and gives the temperature at the middle of the centimeter in column 1. Column 3 is obtained from curve III. (dividing by 60, to bring to seconds), and gives the rate of cooling per second at the middle of the centimeter in column 1. Column 4 is the product of the density, the specific heat, and the rate of cooling. By not using the area of cross-section of the bar, the numbers in this column represent

the heat lost per centimeter length per second by a bar of unit cross-section for the centimeter indicated in column 1. Column 5 gives the flow of heat per second across unit area at the end of the centimeter given in column 1: found by summing the loss of heat per second from all subsequent centimeters of the bar. Column 6 gives the conductivity, in C. G. S. units, at the end of the centimeter given in column 1: found by dividing the difference of temperature at the middle of two centimeters into the flow of heat per second at the cross section between them.

TABLE IV.

RESULTS.

Centimeter of bar.	Mean temperature. (Middle of centimeter.)	Mean rate of cooling. (Middle of centimeter.)	Heat lost per second per unit cross-section.	Flow per second across unit area. (Outer end of centimeter.)	Conductivity. C. G. S. units. (Outer end of centimeter.)
18th	221.0	—	—	1.6154	0.1130
19th	206.7	0.250	0.206	1.4094	0.1060
20th	193.4	0.200	0.165	1.2444	0.1037
21st	181.4	0.162	0.134	1.1104	0.1181
22d	172.0	0.125	0.103	1.0074	0.1259
23d	164.0	0.105	0.0866	0.9208	0.1196
24th	156.3	0.097	0.0800	0.8408	0.1184
25th	149.2	0.085	0.0699	0.7709	0.1264
26th	143.1	0.077	0.0635	0.7074	0.1141
27th	136.9	0.065	0.0536	0.6538	0.1168
28th	131.3	0.058	0.0478	0.6060	0.0962
29th	125.0	0.052	0.0429	0.5631	0.1251
30th	120.5	0.047	0.0388	0.5243	0.1219
31st	116.2	0.045	0.0371	0.4872	0.1037
32d	111.5	0.042	0.0346	0.4526	0.1131
33d	107.5	0.040	0.0330	0.4196	0.0999
34th	103.3	0.035	0.0289	0.3907	0.1184
35th	100.0	0.032	0.0264	0.3643	0.1214
36th	97.0	0.029	0.0239	0.3404	0.1031
37th	93.7	0.027	0.0223	0.3181	0.1060
38th	90.7	0.026	0.0214	0.2967	0.1099
39th	88.0	0.024	0.0198	0.2769	0.0989
40th	85.2	0.022	0.0182	0.2587	0.0958
41st	82.5	0.020	0.0165	0.2422	0.0969
42d	80.0	0.019	0.0157	0.2265	0.0906
43d	77.5	0.018	0.0148	0.2117	0.0960
44th	75.3	0.017	0.0140	0.1977	0.0941
45th	73.2	0.016	0.0132	0.1845	0.0923

TABLE IV. (*continued*).

Centimeter of bar.	Mean temperature. (Middle of centimeter.)	Mean rate of cooling. (Middle of centimeter.)	Heat lost per second per unit cross-section.	Flow per second across unit area. (Outer end of centimeter.)	Conductivity. C. G. S. units. (Outer end of centimeter.)
46th	71.2	0.015	0.0124	0.1721	0.0820
47th	69.1	0.014	0.0116	0.1605	0.0892
48th	67.3	0.013	0.0107	0.1498	0.1152
49th	66.0	0.013	0.0107	0.1391	0.1159
50th	64.8	0.0127	0.0105	0.1286	0.0989
51st	63.5	0.0122	0.0101	0.1185	0.0911
52d	62.2	0.0116	0.0096	0.1089	0.0907
53d	61.0	0.0112	0.0092	0.0997	0.0997
54th	60.0	0.0107	0.0089	0.0908	0.1009
55th	59.1	0.0104	0.0086	0.0822	0.1028
56th	58.3	0.0102	0.0084	0.0738	0.1056
57th	57.6	0.0098	0.0081	0.0657	0.1095
58th	57.0	0.0095	0.0078	0.0579	0.0965
59th	56.4	0.0095	0.0078	0.0501	0.0835
60th	55.8	0.0093	0.0077	0.0424	0.0707
61st	55.2	0.0090	0.0074	0.0350	0.0700
62d	54.7	0.0088	0.0072	0.0278	0.0556
63d	54.2	0.0086	0.0071	0.0207	0.0518
64th	53.8	0.0085	0.0070	0.0137	0.0685
65th	53.6	0.0084	0.0069	0.0068	0.0680
66th	53.5	0.0983	0.0068	—	—

It appears that the results from the last six or seven centimeters of the bar must be rejected. The outer end of the bar (which was too short for so large a diameter) was at a temperature considerably above that of the surrounding air. It seems more probable that this produced an effect along several centimeters of the bar than that there is such a sudden and great change in conductivity between 60° and 50° temperature.

Conclusion.

For such cast iron as was used in this investigation, the thermal conductivity is about fifteen per cent greater from a little below 100° C. to 200° C. than it is between about 60° C. and 90° C.

With a longer bar of less diameter, now in preparation, to be electrically heated, and with a thermo-couple of other materials (perhaps aluminum and silver), I hope to carry this investigation of the thermal conductivity of cast iron to both lower and higher temperatures.

PHYSICAL LABORATORY, PENNSYLVANIA STATE COLLEGE,
June 25, 1894.

On Some Magnetic Characteristics of Iridium.

By S. H. Brackett.

THIS paper does not claim to present all the characteristics of iridium in its magnetic relations, or to discuss the facts to which attention is called, but merely to state some points of interest which seem not to have been noted before, and which suggest reason for further investigation. The work here alluded to was done in intervals of busy elementary science teaching, and may be made more complete when further opportunity occurs.

Iridium was not left out of the list of substances so carefully examined by Faraday, and is mentioned by him as being very slightly diamagnetic. Iridium, as at present manufactured, may be presumed to differ from that used by Faraday, and in the absence of analysis of the specimens here tested there is to be allowed a presumption that they are not pure; but the manufacturer, Mr. John Holland, is authority for saying they are more than 98 per cent pure, with some platinum, a trace of phosphorus and no iron, and no iron has been employed in manipulation.

The first bar tested with such results as to excite further interest in the subject was 13.3 mm. long, 3.2 mm. wide, and 0.9 mm. thick. It was one of several different substances being tested for small amounts of magnetism or diamagnetism. A large electromagnet of portative force of about 5000 grams per square centimeter of surface was furnished with soft iron pole-pieces, shaped so as to furnish a very strong field of force. When the bar of iridium was brought near the poles it was strongly attracted sidewise, and could not be made to stand radially. It acquired a permanent *transverse* polarity so strong as to appear to be a permanent diamagnet. Its extremely small thickness rendered test of attraction and repulsion inoperative.

In the field of any ordinary magnet it everywhere set itself at right angles to the lines of force, and when suspended by a fiber under a glass, it readily assumed the east and west position. In a place where H is determined as 0.158, its period of oscillation was 18.7 seconds, as compared with a steel bar of the same length and weight, whose period in the same place was 5.3 seconds, so that the ratio of its intensity of magnetism to that of steel was nearly as 1 to 12.

It was very difficult to magnetize it longitudinally by the ordinary stroking methods, which were likely to leave it in a very heterogeneous condition, so that it might take a position oblique to the lines of force.

To make further investigation and to determine some of its properties quantitatively, a bar was procured of Mr. Holland, 25.7 mm. long and

3.3 mm. square, weighing 3.57 grams. Apparatus was constructed suitable to the size of this bar, and careful tests were made of its permeability, which was found to be practically unity. Whether the bar had been previously magnetized or had had nearly all magnetism removed by heating, no change of strength of current produced any perceptible change of the lines of force. Even striking it forcibly while it was in the coil failed to produce any magnetism by induction, and it could only be magnetized by actual contact with a magnet and then jarring it.

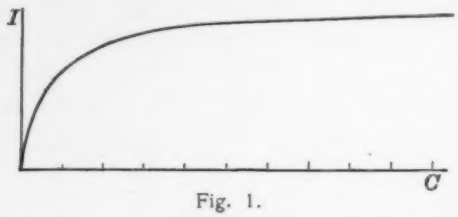

Fig. 1.

On placing it axially between the soft iron pole-pieces of the magnet, and using successively currents of different strength, striking the pole-pieces each time, different degrees of magnetic intensity were given to it, and on plotting the magnetization as ordinates and the current as abscissas, a curve, shown in Fig. 1, was obtained similar to those usually made of permeability.

This bar, as well as the other, is much more readily magnetized transversely than longitudinally. It has only to be placed between flat pole-pieces, so as to be in a uniform field of force, and jarred vigorously. A steel bar of the same size, tempered glass hard, cannot be magnetized transversely when subjected to the same conditions.

When attempts had been made to magnetize it longitudinally, as mentioned above, discrepancies between its magnetic moment at different trials and that obtained by transverse polarity led to further examination of the distribution of its magnetism. The field around it was plotted by means of a little steel magnet 2.5 mm. long, suspended by a cocoon fiber, and the distribution was found to be very irregular. To see if a more uniform distribution could be secured, a method was devised which might be called octuple touch. Stroking was done on all four sides of the bar at once, from the middle to each end. The iridium was held by a small clamp in the center of the space between the poles of the magnet, held only by its center. Four soft iron levers were hinged so that their ends would grasp the iridium near the clamp on one side of it, and four similar levers did the same on the other side of the clamp. These sets of levers were laid on the poles of the magnet, so that when the latter was excited the levers became the

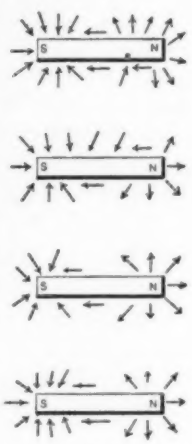

Fig. 2.

real poles of the magnet. When they were applied the circuit was closed, and the sets of levers were held tightly against the magnet, and pulled off in opposite directions at the same time. By this means much better results were obtained, but the distribution was still somewhat irregular, as shown in Fig. 2, representing the four sides of the magnet. The arrows show the exact place of the little exploring magnet.

When magnetized transversely the field of force is much more regular, and it was feasible to delineate this field by sifting filings through a hand-

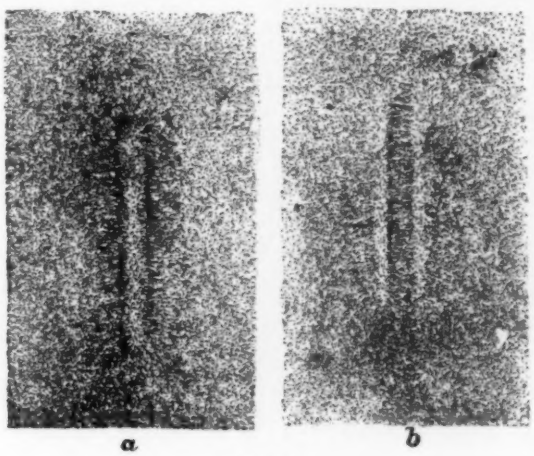

Fig. 3.

kerchief on very thin paraffined paper, fixing them by warming and then photographing. It will be seen (Fig. 3) how plainly the whole side of the bar (*a*) is one pole of the magnet. In (*b*) lines of force are shown crossing the bar, and the usual curves are seen at the ends, which are in this case the real sides of the magnet.

Very low permeability and great coercitive force, with a high intensity of magnetization, are plainly exhibited by these specimens of iridium, and these characteristics are consistent with the known physical properties of the substance.

NOTES.

Hermann von Helmholtz.— On September 8, 1894, at noonday, in Charlottenburg, near Berlin, died Hermann Ludwig Ferdinand von Helmholtz. Few men of his time have been more gifted; few men of science, certainly, have been granted a more universal and generous recognition of their powers. Like Lord Kelvin in Great Britain, Professor von Helmholtz has been for more than a quarter of a century the acknowledged leader of the physicists of his nation.

The fame of Helmholtz rests upon an uncommonly broad foundation. In speaking of him some years ago, the mathematician, W. K. Clifford, who was himself a man peculiarly qualified to form an opinion, said: —

"In the first place he began by studying physiology, dissecting the eye and the ear, and finding out how they acted, and what was their precise constitution; but he found that it was impossible to study the proper action of the eye and ear without studying also the nature of light and sound, which led him to the study of physics. He had already become one of the most accomplished physiologists of this century when he commenced the study of physics, and he is now one of the greatest physicists of this century. He then found it was impossible to study physics without knowing mathematics; and accordingly he took to studying mathematics and he is now one of the most accomplished mathematicians of this century." — *Seeing and Thinking*, p. 18.

Helmholtz, however, was more than physiologist of the first rank, mathematician of the first rank, and physicist of the first rank; he was likewise a philosopher in the best sense of the word, and a writer whose style, considered from the literary standpoint, is remarkable for its clearness and beauty.

One of the main qualities which characterize the scientific work of Helmholtz is a certain completeness and maturity; a completeness which was the natural product of clearness of thought, of deepest insight, and of extraordinary analytical power; a maturity which was the legitimate fruit of the long-continued preparation for his life-work. He was born in Potsdam on the 31st of August, 1821, in which Prussian town his father was a gymnasial master. It was in 1871, as nearly as one may venture to draw the lines which divide a lifetime into its epochs, when, at the age of fifty years, he definitely abandoned physiology for the field of labor with which his name will ever be primarily associated.

Throughout this extraordinarily long preliminary period, during which Helmholtz was successively student of medicine at the University of Berlin (1838–42), assistant at the charity hospital (1842–43), military surgeon in his native city of Potsdam (1843–47), teacher of anatomy in the art academy of Berlin and assistant in the anatomical museum of that city (1848), professor of physiology and general pathology at Königsberg (1848–55), professor of anatomy and physiology at Bonn (1855–58), and professor of physiology at Heidelberg (1858–71), there is abundant evidence from his written work that he was primarily a physiologist, and only incidently engaged in physical investigation. Of the eighty-five papers assigned to him in the Royal Society's catalogue, the greater number are devoted to subjects purely physiological, and in the case of most of the others it is easy to discover that the inquiry was made in connection with some question of physiological importance, or was the outcome of suggestions arising from researches in that science.

Thus in the opening paragraph of the first paper that he devoted specifically to an electrical subject (upon the "Duration and Course of the Electrical Currents induced by Changes in Current Strength," *Poggendorff's Annalen*, Vol. 83, 1851), he says: "The method suggested by Pouillet for the measurement of small time-intervals by means of the action which an electric current exerts during that time upon a magnet, I have used for physiological purposes. In the course of those measurements I found myself compelled to undertake investigations of the duration of the currents induced by sudden changes in the strength of the currents which I had used in the excitation of anatomical parts, and also upon the manner in which the currents used for the measurement of time began, under the influences of the opposed induced currents due to the closing of the circuit, to flow through the coils of the galvanometer. "By means of these experiments I have been able to secure certain confirmations of a very broad mathematical principle, which deals with the determination of the development of such currents. The purely physical features of these investigations are gathered together in the following paragraphs."

Then follows the important monograph familiar to all physicists, in which the limitations of the law of Ohm are indicated, and the case of circuits with fluctuating electromotive forces is treated in the broadest and most fundamental manner.

Even in the case of the essay upon the conservation of energy we note two subordinate articles which would seem to indicate that this great problem was approached from the point of view of the physiologist, and was worked out in the hope of gaining some insight into the mysteries of the vital processes. These two papers, which were among the earliest of the published works of their author, deal respectively with the *consumption of material* and with the *development of heat* in muscular action.

How glorious, even when considered solely with reference to physics, were the fruits of this first period during which physics was subordinated to physiology.

It is marked by a succession of productions, any one of which taken singly would have sufficed to secure its author a lasting place in the annals of the science.

First of all, we have the paper on energy ("Ueber die Erhaltung der Kraft," Berlin, 1847), the strange fate of which it was to be refused admission to the pages of *Poggendorff's Annalen*, to lie well-nigh unnoticed for several years after its appearance in pamphlet form; then to be attacked in vigorous manner by Clausius (1853), and later to subject its author to virulent attacks on the part of Dühring and others, as an imitator of and dishonest borrower from his forerunner, Robert Mayer. At last it came to be recognized as chief of the early attempts to give expression to the principles upon which, later, the doctrine of energy was to be built, and it was reprinted in sumptuous manner as a classic.

This famous essay appears the more remarkable when we consider the year in which it was written, and that it was the performance of a military surgeon in an obscure garrison town, banished for the time from the scientific atmosphere of the universities, and unable, as appears from the reply which Helmholtz made to criticisms of Clausius (*Poggendorff's Annalen*, Vol. 91, 1854), to perform the experiments necessary to the justification of some of his assumptions.

In 1851 followed the description of the ophthalmoscope ("Beschreibung eines Augenspiegels zur Untersuchung der Netzhaut im Lebenden Auge," Berlin, 1851), a discovery which, because of its great utility, served to bring Helmholtz into immediate prominence.

The year 1852 saw the development of the Young-Helmholtz theory of color-sensation. In 1856 appeared the first part of the *Handbuch der Physiologischen Optik*, a work which was to occupy its author for ten years.

The method in which this great work was executed was characteristic of the man. In the preface to the first edition he says, translated freely: —

"It has been my chief endeavor in working out this book to convince myself by the use of my own eyes and by my own experience of the truth of all the facts, even of such as are of minor importance." To carry out such a task in the thorough manner which distinguishes the handbook was in itself a good decade's work, but Helmholtz's mind continued to busy itself with other important problems. In the same year (1856) he published his first acoustical paper, on combination tones; two years later appeared the important monograph upon the hydrodynamical equations which express vortex motion (*Crelle's Journal*, Vol. 55). This was the first of Helmholtz's researches that was clearly free from all physiological

origin, motive, or suggestion. It formed the starting-point of his remarkable series of studies of fluid motion.

In 1859 there was published in the *Münchener Gelehrter Anzeiger*, Vol. 48, also in *Poggendorff's Annalen*, a paper entitled "Die Klangfarbe der Vocale," which contained the germ of what was to be an important chapter in Helmholtz's best known work, the *Tonempfindungen* (1862).

During the years which followed, up to 1871, which year closes the epoch which we are now considering, we find a variety of physiological contributions, the completion of the handbook of physiological optics, and various papers dealing with the topics of electro-dynamics, acoustics, and hydrodynamics, all domains in which the work of Helmholtz had become familiar to readers, and authoritative. Noteworthy also is the opening of a new line of thought. The discussion of the foundations of the Euclidian axioms ("Ueber die thatsächlichen Grundlagen der Geometrie," *Heidelberg Verh. Nat. Med.*, IV., 1868) was a theme to which the philosophic mind of Helmholtz was peculiarly fitted, and his contribution to the theory of the nature of space, which, through the efforts of Grassmann, Riemann, and others, was being developed and established, was of the greatest service.

Such were the principal physical labors of Helmholtz during this long first period — a lifetime in itself, whether measured in years or in achievement. Two other periods followed, perforce shorter, but scarcely less important in their results. One of these may be counted from 1871, in which year Helmholtz went from Heidelberg to Berlin, as successor to Magnus in the chair of physics and as the director of the physical laboratory. It ends with the establishment of the Imperial Institute in 1888. The other comprises the closing years of his life.

The period of his Berlin directorship was one of increased activity in the publication of monographs relating to physics, but it was chiefly notable for the building up of the most important teaching center for original work in Europe.

When Helmholtz was called to Berlin, the laboratory consisted of a suite of rooms in one wing of the main university building. These were abandoned in 1877 for the new laboratory upon the bank of the Spree, a structure unsurpassed, probably, among physical laboratories of that time.

In his relations to the average undergraduate, Helmholtz's attitude was unlike that of some of the other great teachers of science of his day, whose care in the direction of the labors of the youngest inmate of their laboratories was as elaborate as their pains in conducting reseach. Some found him cold and distant, perhaps; but these were, for the most part, of a class to whom he could have imparted but little. To foreigners who came to study with him he was proverbially kind. That he was ever quick to

recognize latent powers and alert to rouse those who possessed them into activity, there is abundant evidence.

Of the long list of important researches which emanated from his laboratory during his directorship, but a small proportion was published under his own name, but those whose privilege it was to make their beginnings in research under his guidance know how great was his skill in selecting a topic adapted to the individual, in outlining a method, and in directing the investigation to a successful ending.

Often his influence upon those who came under him was far reaching in its results. Thus Heinrich Hertz, whose untimely death the world of physics is now mourning, relates in the preface to the work upon which his fame will chiefly rest, that the theme was developed from a problem offered by the Berlin Academy while he was a student in Helmholtz's laboratory; and Helmholtz, on his side, in the introduction which he wrote to Hertz's remarkable posthumous volume on mechanics, recounts the early evidences of power which he discerned in the latter. "Having occasion," he adds, "to propose a theme for a prize essay in physics, I selected a problem in electrodynamics, with a conviction, afterwards confirmed, that Hertz would interest himself in it, and would attack it with success."

This period of double activity, as investigator, with physics as the prime subject of attention, and as director of the most important of university laboratories, drew to a close, as has already been indicated, with the establishment of the Imperial Institute for Physical and Technical Research. To the presidency of this institute at the time of its opening in 1888 Helmholtz was called, and to the direction of investigations, for the pursuance of which it had been founded, he devoted the few remaining years of his life. He had already reached the pinnacle of scientific renown. All the honors — scientific, academic, national, royal, and imperial — which the world had to bestow, had been showered upon him. In 1877 he was rector of the University of Berlin; in 1883, the Emperor William I. bestowed upon him hereditary nobility; in 1891, the celebration of his seventieth birthday, which was held in common with that of his colleague Virchhow, was the occasion of international scientific applause and gratulation.

Up to the time of the stroke which was the near precursor of his earthly end, the powers of Helmholtz showed no signs of waning. Paper after paper in *Weidemann's Annalen* bore witness to his continued scientific productiveness. The number of that periodical for September, 1894, the very month of his death, contained a paper on "Deductions from Maxwell's Theory of the Movements of the Pure Æther"; a memoir quite in his best style, and one the closing lines of which indicated that more might be expected upon the same subject.

The program of the annual meeting which was held in Vienna Sep-

tember 24–30, 1894, contained no less than three papers and addresses which he was to deliver in general session. The second stroke, which came upon him in the first week of September, when he seemed to be recovering from the former attack, cut all his earthly activity short forever.

Much uncompleted work appears to have been interrupted by Helmholtz's sudden death, among other things the completion of the new edition of the *Handbook of Physiological Optics*, and a compendious treatise on Mathematical Physics. The latter book, which is to include the development of optical theory along lines already indicated by the work of Maxwell and of Hertz, will, it is reported, be brought to completion by Dr. Arthur König.

Of Helmholtz's more important monographs, only a few of the earlier have been mentioned, the earlier having been selected, not as more important than his later work, but because they are of especial interest as showing the beginnings in the various lines of thought which were to occupy him throughout the remainder of a long career. Ninety-nine of these were gathered into the first two volumes of his *Wissenschaftliche Ahhandlungen* (1882–83). The third volume is announced to contain all papers subsequent to 1883. The numerous addresses in popular form, delivered by Helmholtz on various public occasions, have likewise been gathered together and constitute the well-known *Populäre Wissenschaftliche Vorträge*.

In 1893, Helmholtz undertook for the first time a journey to America, upon which trip he was accompanied by Mrs. von Helmholtz and by four assistants from the Reichsanstalt. The party visited Chicago, and Helmholtz attended the sittings of the chamber of delegates of the international congress of electricians, of which body he had been elected honorary president. He took an active part in the deliberations of the chamber, to whose service he brought his fundamental knowledge of the subjects under consideration, and likewise a wide experience from the sittings of previous congresses. This was his last important public appearance. On the return voyage it was his misfortune to fall down the companion way of the ship, sustaining injuries from which, at his age, the most serious consequences were to be feared. Scarcely had he made complete recovery when the two paralytic shocks occurred to which his death was due. Thus passed from earth, at the age of seventy-three years, the first physicist of our time.

E. L. N.

The American Association for the Advancement of Science. — The forty-third meeting was held in the Polytechnic and Packer Institute, Brooklyn, New York, August 15–24, 1894. The evening addresses and receptions were held in the Academy of Music and Art Building. The meetings as a whole were well attended, and none of the sections failed to secure an

interesting program. The usual meetings of the various affiliated societies were held in the same buildings, and also the meetings of the Society for the Promotion of Engineering Education; but whether such tend to increase the interest in the meeting of the Association proper or not, is a question. In the intervals between the meetings of the sections, the members of the Association were well entertained by excursions, carefully planned by the local committees, on the Sound, up the Hudson, and down the Harbor to Long Branch; and those interested in the animal life of the deep sea were taken out over night to the Gulf Stream on the steamer *Fish Hawk* of the United States Fish Commission.

Before Section B, Professor William A. Rogers delivered his vice-presidential address upon "Obscure Heat as an Agency in Producing the Expansion of Metals under Air Contract," in addition to which twenty-four papers were presented as follows: —

"A Problem in Stereoscopic Perspective," by W. LeConte Stevens. — "On the Directed Velocity of the Particles in the Electric Arc," by Benjamin W. Snow. — "Aluminum Violins," by Alfred Springer. — "The Photography of Manometric Flames," by William Hallock. — "A New Form of Spectrophotometer," by Edward L. Nichols. — "The Thermal Conductivity of Cast Iron," by I. T. Osmond. — "Thermal Conductivity of Copper at Low Temperature," by R. W. Quick and B. S. Lanphear. — "The Influence of Heat and Electricity upon Young's Modulus for a Piano Wire," by Mary C. Noyes. — "The Influence of Temperature upon the Transparency of Solutions," by Edward L. Nichols and Mary C. Spencer. — "Studies of the Lime Light," by Edward L. Nichols and Mary L. Crehore. — "A Calorific Effect of the Velocity of Migration of Hydrogen Ions," by Samuel Sheldon. — "On the Radiation of Obscure Heat by a Metallic Bar," by Benjamin W. Snow. — "The Infra-Red Spectra of Metals," by E. P. Lewis and E. S. Ferry. — "On Magnetic Potential," by Frederick Bedell. — "On Electric Strength," by George W. Pierce. — "A Phonographic Method for Recording the Alternating Current Curve," by C. J. Rollefson. — "On the Solution-Tension of Metals," by Harry C. Jones. — "A New Recording Thermometer for Closed Spaces," by W. H. Bristol. — "On Some Magnetic Characteristics of Iridium," by S. H. Brackett. — "Lighting as a Case of the Dissipation of Energy," by N. D. C. Hodges. — "Some Peculiar Lightning Flashes," by Alexander McAdie. — "The Attributes of Matter," by J. W. Powell. — "Definition of Motion," by J. W. Powell. — "Device for Comparing Natural with Tempered Musical Scales," by Morris Loeb.

The next meeting will probably be held in San Francisco.

NEW BOOKS.

A Treatise on Hydrostatics. By ALFRED GEORGE GREENHILL, Professor of Mathematics in the Artillery College, Woolwich. pp. 536; 108 figs. London and New York, Macmillan & Co., 1894.

To the engineer, the spectacle of a text-book of more than five hundred pages devoted to the subject of Hydrostatics, by him considered as one of the smaller subdivisions of the Mechanics of Fluids, is something of a surprise. To the mathematician and physicist, however, the case is doubtless different, as Hydrostatics offers a most fruitful and pleasant field for the application of mathematics. That the author of this book shares the latter opinion, and has consequently labored *con amore* in its production, is to be inferred from the following passage taken from p. 3 of the work: —

"Hydrostatics is a subject which, growing originally out of a number of isolated practical problems, satisfies the requirements of perfect accuracy in its application to the largest and smallest phenomena of the behavior of fluids; and at the same time delights the pure theorist by the simplicity of the logic with which the fundamental theorems may be established, and by the elegance of its mathematical operations; so that the subject may be considered as the 'Euclidean Pure Geometry of the Mechanical Sciences.'"

Professor Greenhill is the well-known author of several mathematical text-books and of a portion of the article on Hydromechanics in the *Encyclopædia Britannica* and presents us in the present work with a very full development and discussion of the principles of Hydrostatics and their applications, with some digressions into the field of Hydrodynamics, and a brief presentation of the main elements of Thermodynamics. The usual topics are treated, viz.: Hydrostatic Thrust, Specific Gravity, Floating Bodies, Liquids in a Bent Tube, Pneumatics, Stress in Containing Vessels, Capillarity, Pressure of Liquid in Moving Vessels; and a chapter from Hydrodynamics, called Hydraulics, or Motion of Fluids. While it is evident that the work is designed chiefly for the student of Mathematics and Physics, who will find a wealth of problems (some three hundred or more) of various degrees of difficulty on which to exercise his skill and patience, the author has not neglected the needs of the student of engineering, who will meet with numerous applications to engineering appliances and appa-

ratus, especially in matters relating to Naval Architecture; with, in some cases, detailed descriptions of a most practical and useful character. As instances in point may be mentioned problems, descriptions, and practical matters relating to gauges, safety-valves, barometers, thermometers, caissons, ships, retaining walls, diving-bells, gasholders, siphons, balloons, etc. In many cases certain refinements of treatment are given, not ordinarily presented, such as the stability of a ship when aground, stability of a ship with liquid cargo, barometer with vitiated vacuum, law of increase of the velocity of liquid in a siphon just beginning to operate, etc. The bulk of the book is, therefore, not to be wondered at when we consider its range of treatment and the variety of wants that are met.

Though the use of the Calculus is infrequent, the student is supposed to have a command of its principles and operations, the author remarking in the preface, in quotation marks, "'It is easier to learn the Differential Calculus than to follow a demonstration that attempts to avoid its use.'" In the matter of illustrations, while they are excellent in execution, the reader will find them less numerous than might have been desired and in some instances very inconveniently placed, figure and corresponding text being sometimes several pages apart. Occasionally a single figure, crowded with the necessary detail, is made to serve for several different demonstrations or references. It must be said that the letterpress has a rather monotonous appearance, which would have been much relieved, and the reader aided, by the use of heavy type for paragraph headings and leading substantives. Numerous references, scattered throughout the work, form quite an ample bibliography of the subject.

Various commendable features of the book, as seen from the standpoint of the engineer, are the following: (1) the frequent insertion of numerical examples and illustrations; (2) the use of the gravitation unit of force, as pounds, kilograms, and tons; (3) the definite expression of the nature of a quantity denoted by a symbol, or combination of symbols, as secured by adding the full statement of its character in some system of units, such as P pounds, v feet per second, A square feet, p pounds per square inch, D pounds per cubic foot; these expressions, however, being abbreviated, respectively, to P lbs., v ft./sec., A ft.^{2}, p lbs./in.^{2}, D lbs./ft.^{3}, which are just as effective; (4) dealing with the specific gravity of a substance chiefly by employing the weight of a unit of volume, *i.e.* its "heaviness" or "density." In this connection it is worthy of note that although the author occasionally asks the student to determine the whole normal fluid pressure on a *curved* surface, he makes prominent the fact that this sum total has no mechanical significance (p. 82). The obtaining of this total normal pressure is, of course, little more than an exercise in mathematics, but in some books, from failure of their authors to mention the above fact,

the student is led to think that the total normal pressure is of some practical importance.

Further comment is suggested by a few incidental points. On p. 8, where stress is defined as composed of two "equal and opposite balancing forces," the word "balancing" seems of doubtful propriety, since two or more forces can hardly be said to balance each other unless they all act on the same body, and that body is in equilibrium under their action; and even then the phrase does not seem strictly appropriate, since force is the name given to actions of bodies on bodies, we having no conception of an action of force on force. Again, on p. 8, as to the quotation from Newton's third law of motion that action and reaction "are equal and opposite," it has always seemed unfortunate to the present writer that the word "simultaneous" is not usually added, in order to forestall misconception, since in popular and historical language the "reaction" is generally subsequent to the "action." On p. 21, the "Keeley Motor in America" is referred to in a way that seems to lend it more dignity than it deserves.

The comparison suggested in Ex. 11 of p. 185 evidently refers to the Principle of Least Work used in dealing with structures having redundant elastic members. The solution of the problem of finding the stresses in a thick hollow cylinder (p. 395) has the same unsatisfactory character as the treatment in Rankin's *Applied Mechanics*, the student being left in doubt as to the certainty of the result; however, a final test of its correctness could not well be supplied, as is done in Cotterill's *Applied Mechanics*, without some groundwork of the theory of elasticity.

The "water ram," or increased pressure in a pipe in which water is flowing, occasioned by the shutting of the escape valve in a given time, is mentioned on p. 428, and an average value (a time-average) determined. Such an average value, however, is of little practical value, as the maximum pressure occurring during the closing of the valve may be enormously greater than the average, this maximum depending not simply on the total time of closing, but upon the rate of closing at different stages of the operation, as well as, to some extent, on the compressibility of the liquid, and the extensibility of the enclosing pipe.

In the treatment of the hydraulic ram (p. 428) it is difficult to understand why the author should have resorted to the very roughly approximate method there given, instead of using the more satisfactory one already employed in the case of the siphon (when first set into operation [unsteady flow], see p. 274). According to the expression derived on p. 428, the efficiency is greater the greater the height to which the water is lifted, with a given working head, or fall; just the reverse of actual fact, as shown by Eytelwein's experiments.

The hint, on p. 438, that a certain preceding investigation might serve

to determine, to some extent, the tendency to produce sea-sickness at a given point of a rolling ship or swinging body, suggests to the present writer the propriety of calling attention to a somewhat similar case where the motion is rectilinear, as in a railroad car on a straight horizontal track. So long as the velocity is uniform, whatever its amount, a person standing in the car, having once adjusted his feet for relative stability of his body, feels no need of further adjustment. If now the velocity, v, begins to increase, and continues to increase with a *constant* acceleration, p, after a single readjustment of the feet at the instant of this change of acceleration (which was also *constant*, =zero, while the velocity was uniform), no necessity is felt for further adjustment. From this we may conclude that the tendency to sea-sickness — that is, the frequent necessity of readjustment of support to preserve relative equilibrium — depends, not on velocity, nor even on acceleration, but on the time rate of *change* of the acceleration; that is, on the value of $\frac{dp}{dt}$, or $\frac{d^2v}{dt^2}$, or $\frac{d^3s}{dt^3}$; where s and t denote distance and time respectively. The more abrupt and frequent the changes in the acceleration, the greater the disturbance of relative equilibrium and the disquieting physiological effect dependent thereon. The occurrence of a normal acceleration, due to the passing of the car from a straight to a curved portion of the track, calls for similar readjustments of support laterally, and the railroad engineer makes use of a "transition curve" (or curve of gradual change of curvature) to secure a gradual instead of an abrupt change of normal acceleration.

In the chapter on Hydraulics, or Motion of Fluids (p. 461), by a remarkable oversight the author makes no mention of steady, as distinguished from unsteady, flow; cases previously considered of liquids in motion (hydraulic ram, and the siphon in initial stages) belonging to the latter category, while those of this chapter are comprehended in the former. The chapter contains about twenty pages and is from necessity very tersely written, the topics considered being beyond the strict scope of the book; but contains frequent numerical illustrations. Occasionally a theoretical result is announced without sufficient mention of practical facts or limitations. For example (p. 464), it is stated of a Pitot tube (a "J-shaped tube"), that if the extremity of its shorter upturned branch receive the impact of the falling water at the base of Niagara, the height of the fall being 162 feet, a column of stationary water of double this height, *i.e.* 324 feet, will be balanced in the other branch. It would have been well to give some experimental results in this connection. With a Pitot tube of small bore and sharp edge at the point presented to the current, Darcy found the height, h, of still water maintained in the vertical tube almost exactly equal to the height due to the velocity, v, of the stream; *i.e.*

$h=v^2/2g$. However, with one of his own instruments, probably furnished with a very blunt point at the extremity, Weisbach obtained $h=\frac{1.56\,v^2}{2g}$, with velocities ranging from 0.32 to 1.24 meters per second. In the Civil Engineering laboratories at Cornell University, results for this height, h, have been found to differ but little for Pitot tubes of three different forms of tip: one sharp-edged like that of a pencil case; another with the end faced off square, the wall of the tube having a thickness equal to the diameter of the bore; the third having a flaring or trumpet-shaped extremity. The bore of the tube was the same in all three cases; viz., 0.042 in., while the velocities ranged from 6 to 18 feet per second. The formula obtained was $h=\frac{1.2\,v^2}{2g}$, nearly.

To the three laws of fluid friction (p. 479) might have been added a fourth: that it is proportional to the "heaviness" of the fluid; and on p. 480 attention might have been called to the fact that of the two coefficients there used, f is an abstract number, while k is not. The assertion on p. 480, as to loss of energy due to fluid friction in a pipe (with steady flow), that if $\frac{d}{l}=4f$, all the energy due to the head $\frac{v^2}{2g}$ is wasted by frictional drag, seems of little importance, since without further data no means are at hand for determining the velocity, v.

At the foot of p. 480 (though attendant circumstances are not made very specific), we find a brief proof of a fact mentioned by Professor Cotterill (*Applied Mechanics*, § 259), communicated to him by Mr. Hearson, that in the case of a long supply pipe and an unlimited water supply, the speed of a motor at the extremity should be regulated to such a value, for a maximum efficiency, that the loss of head from fluid friction in the pipe is one-third of the total head. On p. 463 we find a definition given of the "Miner's Inch," as if it were a fixed module; whereas different values are used in different localities in the mining states of America.

I. P. CHURCH.

Schneekrystalle Beobachtungen und Studien. Von Prof. Dr. G. HELLMANN. Mit elf Abbildungen im Text und acht Tafeln in Heliogravure und Lichtdruck nach mikrophotographischen Aufnahmen von Dr. R. Neuhauss in Berlin. Large 8vo, 66 pp. Berlin, R. Mückenberger, 1893.

It is rare to find a work which even in its outward form bespeaks the beauty of the subject of which it treats, as does this dainty volume on snow crystals by Professor Hellmann. With its silver-lettered, pale gray binding, its many beautiful illustrations, and its excellent typographical work, it makes a most attractive appearance.

As will be seen from the title, this work is the result of observations and studies of snow crystals by the author. For many years he had busied himself with the examination and study of these dainty crystallizations. Again and again he attempted to draw by hand the ever-varying forms as he beheld them through a microscope, only to find, however, that an exact reproduction in this way was utterly impossible, and that all pictures of snow crystals that had ever been given, as, for instance, we find them in our text-books of physics and meteorology, were more or less conventional. Finally he was so fortunate as to secure, in the winter of 1892 and 1893, the co-operation of Dr. Neuhauss, whose achievements in photography are well known. With Dr. Neuhäuss's aid, Professor Hellman has been able to give to the world the first microphotographs of snow crystals ever printed, and hence this book brings us the first true pictures of these evanescent, flimsy structures, as they are found on the surface of the earth. It is needless to say that this is a most praiseworthy achievement. The eight plates given, of which one is photogravure and the others heliotype, have been reproduced from Dr. Neuhauss's microphotographs, the magnification being twelve and twenty times the natural size of the crystals. The plates permit the use of a lens, so that even minute details may be examined with ease.

The book is divided into four chapters, the first, historical, being entitled: The Study of Snow Crystals from the Time of Albertus Magnus to James Glaisher. This chapter contains the most noteworthy cuts of snow crystals found in the literature of this epoch (1555 to 1855), and thus affords the opportunity for a most instructive examination. The last ones, those of Glaisher in 1855, are those usually found in our text-books. When these are compared with the microphotographs, the first feeling is likely to be of disappointment; for the beautiful and wonderful symmetry which we were accustomed to see, has disappeared, or very nearly so, and instead appear asymmetrical structures that are but rarely complete in form. But it is this very fact which gives value to these photographic reproductions. Here we have not what the eye has not seen and the hand been tempted to add. Instead of cold, ideal pictures, we have warm, living structures, whose very mutilations reveal the life within them. The hexagonal formation underlying all snow crystals was first pointed out by Kepler in 1611.

The second chapter deals with the Morphology of Snow Crystals. But brief mention can be made here of the salient points. Of the sixty snow figures given on the eight plates but four were found upon accurate measurement to be perfectly regular, one of them being a six-sided laminar one with a diameter of but 0.3 mm. The deformation in outward form is probably due to air motions and to the resistances which the flake encounters on its downward journey. When these disturbing conditions are a minimum, — that is, on a perfectly quiet day, — the most regular forms occur.

These deformations can be classified as imperfect formations, displacements, and asymmetries, all revealed by the plates, and even capable of measurement. With respect to the inner structure of the snow crystal, the most remarkable circumstance is that in all forms appear pointed capillary tubes filled with air or water. This fact serves to differentiate the snow crystals from all other ice crystals, such, for example, as occur in window-pane frosting, where the author failed to find these tubes. Nordenskiöld even observed these tubes filled with water when the temperature was −8° C. As to the size of the crystal, it was found that the star-shaped forms are always larger than the laminar ones. The mean diameter of the former was 2.4 mm., of the latter 1.3 mm. The largest occurring was 7–8 mm.; the smallest, 0.3 mm. The average size of the more rarely occurring six-sided prismatic forms could not so well be ascertained. The dependence of size upon temperature is as follows: The mean diameter of radiating, star-shaped crystals at −6° C., was 3.4 mm.; at −8°, 2.2 mm.; and at −12°, 1.2 mm., decrease in size thus accompanying decrease in temperature. This is due to the fact that with decrease of temperature the amount of aqueous vapor in the atmosphere decreases, and that, too, almost in the same proportion exhibited by the decrease in the size of the crystals. The number of star-shaped forms decreases as the temperature decreases, while the thin laminar ones increase in number.

In the third chapter, New Division of Snow Crystals, the following classification on a crystallographic basis is proposed: 1. Tabular snow crystals, as, for example, those where the main axis or thickness of the crystals is very small in comparison with the other and secondary axes, such as star-shaped forms, plates, and combinations of these two; 2. Snow crystals columnar in form, *i.e.* where the development has been nearly even in the direction of each axis. About 75 per cent of all snow crystals are to be referred to the first class.

The last chapter, headed Origin of Snow Crystals, is a very short one, and reveals how little we know about these interesting formations. It is generally believed now that they are formed directly from the aqueous vapor in the atmosphere without first going through the state of liquefaction. They begin probably as microscopical nuclei of snow which occur frequently in the upper regions of the atmosphere and in the polar regions. Around these nuclei as a center the snow crystal is gradually evolved by new crystallizations. But such questions as to the nature of the inner molecular forces which come into play, or the cause of the different hexagonal shapes which we meet with in the same snow-fall, or the conditions under which Class I. or Class II. are formed, have as yet received no answer. Thus to the physicist and meteorologist are suggested interesting questions for solution.

Professor Hellmann closes his valuable contribution to science with a bibliographical appendix containing notes and extracts from the works cited.

L. A. BAUER.

Recent Text-Books in Heat.[1]

Of the four books reviewed below, the most extensive is the one by Mark R. Wright. This book is of an intermediate grade, between beginning work and advanced work. According to the preface it "is intended for those who have read the elementary parts of the subject as treated in 'Sound, Light, and Heat,' or who are able at once to attack a more advanced work."

The author has attempted to cover all branches of the subject generally treated under that head, including a chapter on Thermodynamics, and one on Climate. The subject of the heat produced during chemical combination has been entirely omitted. The omission is probably intentional, but it would seem that there is no good reason why this subject should be entirely neglected in a work of this character.

Such a large portion of all the important work in heat has been described, or at least mentioned, that one is tempted to look for omissions. With this in view, we find that although Langley's bolometer is described, no mention is made of Boy's radio-micrometer. No mention is made of that very promising advance in calorimetry, known as Joly's steam calorimeter. Siemen's "electrical pyrometer" is described, but nothing is said of the interesting work of Callender and Griffiths on platinum resistance thermometers.

We are a little surprised to find Rowland's researches on Joule's equivalent passed with a sentence. The statement is made that Rowland "obtained numbers which indicate that J is not constant." We should expect this statement to be followed by an explanation of the effect of these researches on our knowledge of the specific heat of water at different temperatures.

It is unfortunate that the number 772 should be retained for Joule's equivalent, although it has been proved that Joule's own work, when properly corrected for the specific heat of water, gives a considerably larger number.

[1] Heat. By Mark R. Wright. pp. 336. New York, Longmans, Green & Co., 1893.

Practical Work in Heat. By W. G. Woollcombe. pp. 61. Oxford, Clarendon Press, 1893.

Heat: an Elementary Text-book, Theoretical and Practical. By R. T. Glazebrook. pp. 230. Cambridge, University Press, 1894.

Thermodynamics of Reversible Cycles in Gases and Saturated Vapors. Lectures by M. I. Pupin. Edited by Max Osterberg. pp. 114. New York, John Wiley & Sons, 1894.

In speaking of the heat absorbed by the atmosphere, Poulet's work is referred to, but nothing is said of Langley's Mt. Whitney researches. On p. 95, we are told that "of all known substances water has the greatest specific heat." This is clearly an inadvertence, as the specific heat of hydrogen is correctly given in another place.

The book is written from the standpoint of a teacher. It is divided into thirteen chapters, and they are subdivided into paragraphs with headings in full-faced type. The presswork is excellent, as are the numerous illustrations, quite a number of which are new.

The description of experiments is good, and the explanation of principles is unusually satisfactory. The derivation of formulas is very complete, sometimes superfluously so. The author seems to have made an especial effort to help the student over his greatest difficulty, namely, the translation of physical laws and principles into formulas and numerical relations. To this end one finds everywhere general laws and principles, as well as experimental facts, put into the form of equations in a manner that no student ought to find abrupt. Accompanying this work are numerical illustrations and solved problems in profusion. The completeness of this part of the work makes it especially desirable as a reference book, or for students reading the subject without a teacher. As a text-book for the class-room, teachers will be disposed to think that it leaves too little for the student to do for himself.

There is one thing, however, in this numerical work that ought to be protested against. Numbers and results are often given to the fourth, fifth, and sometimes to the sixth and seventh significant figure. Any one who has done experimental work in heat knows that the fourth figure is seldom reached even by the skilled experimenter. It is surely then a waste of energy, as well as positively misleading, to carry numerical calculations so far. One of the hardest things to instill into the practices of laboratory students is the proper degree of refinement to be used in their computations.

In the preface the author states that, "A fair selection of the experiments described should be performed by the student." The book is, nevertheless, designed for the class-room, and experiments are described more as work that has been or may be done, than as work to be done.

Woollcombe's *Practical Work in Heat* is a small volume devoted wholly to laboratory work. In its 60 pages, 50 experiments are described under the heads Thermometry, Expansion, Calorimetry, Evaporation, and Radiation. The work has grown out of lectures and manuscript directions to a class of boys in an English High School. Although most of the experiments are "within the range of boys of fourteen," they are not so elementary as that would make it appear. All the experiments are quantitative, *i.e.* none are purely illustrative.

The directions for performing experiments are concise, though full enough for the purpose. The formulas are derived in full, leaving nothing for the student to do but to substitute his observed quantities in them.

This course of experiments is taken consecutively by the whole class, which necessitates that the apparatus be simple and inexpensive. Much of it we are told "may be readily obtained or easily made from the ordinary stock of a chemical laboratory."

The author very properly emphasizes the necessity of having the students tabulate their data and results. To this end forms are given to show the student how best to arrange his work. It would have added to the value of these forms if they had contained typical data of no greater degree of accuracy than the best students might attain under favorable circumstances.

Very little is said of sources of error or how to avoid them.

The author seems to have lost sight of the fact that small errors of observation may lead to a relatively much greater error in a result, when the latter is obtained by elimination between two equations. In Experiment 39, for example, both the heat of fusion and the specific heat of paraffin are determined by elimination between two equations involving observed quantities and the two unknowns. As each of these equations is already a function of difference of temperatures, the result obtained will frequently be of little value. Another example, on p. 59, is one in which a bulb tube is calibrated by weighing it empty, and twice when containing mercury to different heights in the tube. It would be very much more accurate to weigh directly an amount of mercury which will fill a given length of the tube, and also the bulb full of mercury. It is difficult for a student to understand why results obtained as in the examples given are often so far wrong, even when the observations have been carefully made, and the mathematical work is correct.

Glazebrook's elementary text-book on Heat belongs to the "Cambridge Natural Science Manuals." It is at once a laboratory manual and a text-book for the class-room. The book is of quite an elementary character, although it covers a great deal of ground, and one who thoroughly grasps what it contains knows a good deal about heat. Quite a number of the 55 experiments to be performed by the student are purely illustrative, although the greater portion of the work is quantitative.

The experiments are performed by the student in connection with a text-book course of study which is taken at the same time. The object of the experimental work is merely to give the student a real knowledge at first hand of the science he is studying, and not to make him a skilled experimenter. Each student performs a large number of experiments with simple apparatus, and is not expected to attain any considerable degree of accuracy.

In this connection one thing to be highly commended is the specific pointing out of sources of error under the head "Defects of the Experiment," or "Sources of Error." It seems to the reviewer, however, that the author has left out the most prolific source of error in calorimeter work; namely, the errors affecting temperature determinations.

Both Woollcombe and Glazebrook have been eminently successful in arranging a series of quantitative experiments in heat which do not require expensive apparatus or skillful manipulation. However, it must be remembered that the results attained cannot be *very* much better than qualitative ones. The probable error will, in most cases, be very large.

Teachers will find in both these books some new experiments, old experiments performed with simple apparatus, and many valuable suggestions on the construction and setting up of such apparatus.

Browne's translation of Clausius's work on Thermodynamics has now been out of print for some time. This fact lends an especial interest to Dr. Pupin's lectures, which have been edited by a student in Columbia College. These lectures were delivered in a ten weeks' course to a class of undergraduates. "The course forms a theoretical introduction to a practical course on heat engines." On account of the brevity of the course, the author confines himself to reversible cycles in gases and saturated vapors. The methods and notation of Clausius are frankly adopted. Indeed, the lectures consist almost wholly of judicious selections from Clausius of those parts most interesting to engineering students. We have first a statement of the first law of thermodynamics and its application to perfect gases. Next the second law of thermodynamics is derived from the "Axiom of Clausius," and finally the two laws are applied to determine the behavior of saturated vapor during reversible operations.

Much of what we find in Clausius's classical work is omitted, as unsuitable to the purpose in view. On the other hand, some of the mathematical expressions which Clausius has left to the student to interpret we find here quite fully explained. This is notably true in regard to the meaning and derivation of the differential equation $\frac{dQ}{T} = dS$, from the integral equation for cyclical operations, $\int \frac{dQ}{T} = 0$. This is a point which has been in great need of explanation, students generally finding Clausius's treatment inadequate. I think Dr. Pupin's treatment capable of considerable improvement, although it is quite clear as it stands. In this connection, I would call attention to a misstatement. On p. 48 we find this statement: "In other words, dQ_{12} [an infinitesimal quantity of heat passing in or out of the working substance] depends on the volume, pressure, and temperature, and the method of their variation during the operation represented

by the element 12. Now all these things are completely defined by the co-ordinates of the extremities of the element, *hence* dQ_{12} *is a function of these co-ordinates.* The same is true of the infinitely small quantity $\frac{dQ_{12}}{T_{12}}$."

If dQ_{12} is strictly a function of the initial and final conditions of the working substance, so must it be when the quantity of heat becomes finite, and this is not true. Probably it is meant that this is true within an infinitesimal of the second order. Even in this light, the above reasoning is at fault, for the quantity $\frac{dQ}{T}$ is solely a function of the initial and final conditions, no matter whether the quantity of heat is finite or infinitesimal.

Another case in which the author explains quite fully what is sometimes a little obscure, is the specific heat of a saturated vapor. For this purpose, he invents a "definitional process" which the student cannot fail to find a great help in understanding what is meant by the specific heat of a saturated vapor.

On the whole, these lectures contain, in small compass, a very satisfactory treatment of the thermodynamics of gases and saturated vapors. One cannot help wishing, however, that the author had edited his own lectures.

F. J. ROGERS.

A Laboratory Manual of Physics and Applied Electricity. Arranged and edited by EDWARD L. NICHOLS. Two volumes. New York, Macmillan & Co., 1894.

The first volume of this Manual has been prepared by Ernest Merritt and Frederick J. Rogers, and constitutes a Junior course in general Physics. The work is arranged as a series of separate experiments, arranged in groups for student instruction.

The second volume consists of Senior courses and outlines of advanced work arranged in four parts as follows: Part I., "Experiments with Direct Current Apparatus," by G. S. Moler, H. J. Hotchkiss, and C. P. Matthews; Part II., "Experiments with Alternating Currents," by Frederick Bedell; Part III., "Senior Courses in Heat and Photometry," by C. P. Matthews; Part IV., "Outlines of Advanced Work in General Physics," by Edward L. Nichols.

The Incandescent Lamp and its Manufacture. By GILBERT S. RAM. London, The Electrician Publishing Company, 1894.

In the review of this work which appeared in the PHYSICAL REVIEW, Vol. I., No. 2, the name of the author was incorrectly given as S. T. Ram.

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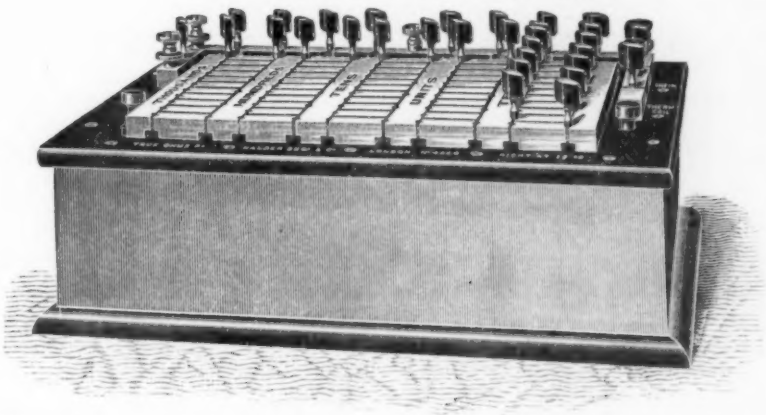

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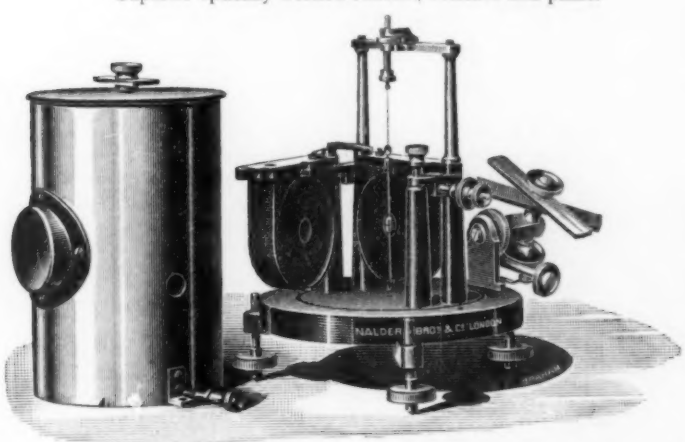

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